## THE ART AND SCIENCE OF LUNAR GRAVITY ASSIST

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#### ABSTRACT

This paper contains elements of design and analysis for cislunar missions that make use of the lunar gravity to shape the trajectory and to enhance the scientific return. The elements of design are the artistic considerations because they require the integration of diverse twobody, three-body, and four-body concepts in a complex and non-intuitive environment. The discovery of the Double Lunar Swingby is cited as an elegant example. The analytic elements are the methods applied to the design concepts that permit the verification of feasibility and the design of real-world missions. These elements are discussed in the context of some preliminary mission design concepts for the ISTP cislunar missions along with some suggestions that may further enhance the flexibility and scientific return of such missions.

#### INTRODUCTION

The International Solar-Terrestrial Physics (ISTP) program is the first mission in history to make extensive use of the great strength of the lunar gravity to achieve objectives that would otherwise be so difficult as to appear untenable. The dynamical heart of the program is the use of the Double Lunar Swingby (DLS) technique that permits repeated traverses of the geotail and the region sunward along the Earth-Sun line containing the bow-shock caused by the interaction of the Earth's magnetic field and the impinging solar wind. In the ISTP mission profile, the ESA spacecraft SOHO and the U.S. spacecraft WIND monitor the sunward phenomena while the Japanese spacecraft GEOTAIL monitors the activity in the anti-sunward direction. Both WIND and GEOTAIL make extensive use of the DLS technique while SOHO monitors the solar input from an "orbit" about the interior Sun-Earth collinear libration point where it is joined by WIND toward the end of the dynamically active part of the mission. CRRES , POLAR, and CLUSTER monitor activity near the Earth.

Because the ISTP program makes such extensive use of lunar gravity assist, it is important to study the methods of gravity-assisted orbit design and to identify those regions in cislunar space where the various approximate methods are valid and where they are not. In this way, one can gain a "feeling" for what kinds of transfer orbits are possible and the regions of motion where more detailed perturbation analyses will be required to establish feasibility. This

paper contains discussions of the zero-sphere-of-influence (point-to-point) patched conic and the use of the Jacobian integral both in the Earth-Moon system and in the Sun-Earth/Moon system. These considerations are used, for example, in determination of Earth-to-Moon transfer trajectories that result in certain desired DLS orbits after the first lunar encounter and how the translunar orbit inclination can be adjusted to yield the desired post-swingby orbit.

Also included are less analytic discussions of the problem of targeting through a lunar swingby to a low- $\Delta V$  insertion into a quasi-stable "halo" orbit about the L<sub>1</sub> (interior) libration point of the Sun-Earth/Moon system. Semi-analytic discussions show the path to finding gravity-assisted transfers that require less than 50 m/s for halo orbit insertion. Finally are some suggestions for lunar gravity-assisted maneuvers that permit the rapid transfer from sunward to anti-sunward DLS orbits. A high-inclination, near circular moon-to-moon transfer orbit is shown to permit great flexibility in planning cislunar missions for maximum scientific return. It is emphasized that there are probably many as yet undiscovered maneuvers that can be used both for scientific exploration and commercial operations in the Earth-moon system and that, while a knowledge of the principles of orbital mechanics is essential for their discovery and application, a healthy imagination and a sense of "seat-of-the-pants" mission design is often helpful. Such an attitude, when tempered by a return to real-world mission constraints and trajectories, adds a pleasant poetry to orbit design.

#### THE DOUBLE LUNAR SWINGBY

The Double Lunar Swingby<sup>1</sup> (DLS) technique is a clever and very useful application of gravity assist trajectories that maintains the spacecraft's line of apsides along a direction fixed with respect to a 4th body about which the three-body system is in orbit. The DLS concept is one which is "obvious" once stated but one which requires the integration of a number of factors in a dynamic environment of what appears at first thought to be great complexity. The discovery of the DLS method, then, has not only a valuable analytic or scientific character but also a rewarding artistic nature that sets it apart not just as a useful exploration tool but as a discovery of great beauty. It is important both esthetically and practically to examine the elements that go into such discoveries if we hope to make other advances in the future. The analytic elements of such an examination are the inputs available to the discoverer, his or her training and level of proficiency in the subject and related technologies, an ability to discriminate between the important and unimportant, and the availability of tools and methodology to study the problem. The artistic elements are not so easy to identify and are often poo-pooed in today's ultra-conservative engineering business environment. They are, in the author's experience, a persistent curiosity, a sense of freedom, a sense of urgency or desperatness particularly if brought on by the imminent need for or usefulness of a solution, and a certain "bulldog" characteristic that allows the innovator to return time and time again to the apparent chaos of the problem. A more general characteristic of discovery is the ability of the innovator to examine the problem in a new perspective or from a different point of view. It is very likely that this characteristic played a major role in Farguhar's discovery of the DLS trajectories.

One of Robert Farquhar's trademarks , sometimes to the dismay of his colleagues, is his propensity for showing trajectories in a rotating frame, usually choosing the x axis as the line joining two of the primaries in the restricted 3 or 4 body problems. This is not just an ideosyncracy; Farquhar actually thinks in rotating coordinates. This is shown by his phraseology in Ref. 1 where he describes Egorov's<sup>2</sup> general investigation of lunar-swingby trajectories:

# " ...Egorov showed how the moon's gravity could be used to oppose the natural rotation of the apsidal line."

The phrase "natural rotation of the apsidal line" is the giveaway that we must be reading a paper by Farquhar. Only in a rotating coordinate frame can the inertially fixed direction of the apsidal line be considered a "natural rotation." In other references to this "rotation", Farquhar is careful to qualify the terminology, e.g. " ... the natural orbital precession with respect to the sun-Earth line will ...." but in the phrase above, he expects the reader to be with him in the rotating frame as he finds ways to "... avoid the unwanted apsidal rotation ..."

Thus Farquhar had all the artistic or "right brain" elements listed above as well as the analytic or "left brain" characteristics necessary to articulate and implement his discovery in a real-world mission scenario. We shall return to the artistic realm of lunar gravity assist later in the paper, but before switching to the very analytic sections to follow, it may be of interest to examine one more artistic aspect of the DLS. The present author took exception to only one statement of Ref. 1. The statement was that the DLS technique was "fundamentally different from gravity assist concepts formulated in other studies." The exception was made on the purely analytic grounds that all the fundamentals used in the DLS had previously been used in Jupiter orbiter studies<sup>3</sup> including orbit turning to place the apoapsis in the Jovian bow-shock. On rereading the paper, the author wishes to remove the exception to the statement on esthetic grounds. The offending statement was followed by a reference to the fact that the DLS trajectories are doubly periodic and sun-synchronous. Examination of Fig. 6 of Ref. 1 will show that the DLS trajectories are symmetric not only with respect to the Earth-sun line but also with respect to the Earth-Moon line. This embedded symmetry is very pleasing and almost certainly useful for some (as yet) unknown future application. The DLS technique *is* fundamentally different from other gravity assist concepts - - it is more profound and it is fundamentally much more beautiful.

#### **GRAVITY ASSIST IN THE EARTH-MOON SYSTEM**

The use of gravity-assist or planetary swingbys in space exploration has an impressive history including early lunar swingbys to escape the Earth-Moon system, the exploration of the outer planets by Pioneer and Voyager spacecraft, and the remarkable trek of ISEE-3/ICE from L1 to the geotail and on to the comet Giacobinni-Zinner. This latter demonstrated, beyond any doubt, the usefulness and real-world feasibility of cislunar orbit control using lunar gravity assist. The methods by which these orbital transfers can be analyzed are referred to as "patched" or "matched" conics in conjunction with solutions to the problem of Lambert wherein the spacecraft is presumed to transfer from a point on one orbit to a point on the Moon's orbit in a given time. The energy relationships are very accurate in these methods but the actual timing and passage distances are difficult to predict. In the following, the analysis is confined to discussions of the orbital parameters well before and well after a particular swingby. Only after these relationships are established are the details of the actual swingby examined to determine feasibility.

These preliminary studies have been confined to the framework of the circular restricted threebody problem ignoring the ellipticity of the moon's orbit and the (considerable) influence of the solar gravitational perturbations on the motion. While these important factors are accounted for in the numerical examples presented later, these calculations are accurate enough in terms of pre- and post-swingby energy and (therefore) timing relationships to provide a firm foundation for the numerical integrations that account for all important details of the actual trajectories. The nature of the mission objectives for the ISTP program and the great strength of the lunar gravity open up mission design and launch strategy options that are not usually available to the mission planner. For example, the fact that the desired trajectory includes two separate Earth orbits permits the consideration of two distinct launch opportunities each month; the initial Earth-to-Moon transfer orbit can be targeted either to the inbound or outbound encounter of the Double Lunar Swingby (DLS) orbit. Furthermore, the orbit shaping power of a single lunar swingby allows the use of transfer trajectories that are highly inclined to the Earth-Moon plane, an option not easily available in the design of lunar orbit missions because of an increase in the Moon-relative energy for out-of-plane transfers. For the WIND and GEOTAIL missions, the higher energy with respect to the Moon is actually desirable because it increases the power of the lunar swingby to change the orbit of the spacecraft with respect to the Earth.

The following paragraphs contain discussions of the fundamentals of gravity-assist trajectories as they apply to the cislunar mission design; the use of non-coplanar trajectories which always allow transfer using a due-East launch from ETR; and the geometry of the Earth-Moon system in 1992. It is emphasized throughout that the variety of Double Lunar Swingby orbits provides a wide range of translunar targeting options, within two distinct launch opportunities each month, that add an unprecedented flexibility to launch and mission operations planning.

A single lunar swingby on a translunar trajectory that barely gets to the moon can add sufficient energy that the spacecraft escapes the Earth-Moon system. The difference between the size of the two orbits of the [1,1,3] DLS is evidence of the great strength of the lunar gravity for these applications. The analytical details of these trajectories are well-documented (see e.g. Refs. 1, 3, and 4). What is most important to the analysis presented here is the relationship between the orbital energy and axial angular momentum as given by the famous Jacobian integral of the circular restricted problem of three bodies.

Jacobi's integral can easily be written in a non-rotating coordinate system as  $C = E - n' h_Z$  where E represents the Earth-relative orbital energy of the particle or spacecraft, n' is the orbital mean motion of the Moon, and  $h_Z$  is the component of spacecraft orbital angular momentum normal to the plane of motion of the two primary bodies. In terms of more familiar orbital elements, the integral can be written

 $C = -\mu/2a - n' [\mu a(1-e^2)]^{1/2} \cos i.$ 

Here,  $\mu$  is the gravitational constant of the central primary (Earth), a and e the semi-major axis and eccentricity of the orbit, and i is the inclination of the orbit with respect to the Earth-Moon plane. This relationship is particularly useful for design and study of Double Lunar Swingby orbits because, in the circular restricted problem, the value of C must remain constant no matter how many times the spacecraft encounters the Moon.

Furthermore, in any one swingby,

$$\Delta \mathbf{E} = \mathbf{n}' \, \Delta \mathbf{h}_{\mathbf{Z}}.$$

This simple relationship means that the change in Earth-relative energy caused by any lunar swingby will be accompanied by a change  $\Delta E/n'$  in the component of angular momentum that is normal to the Earth-Moon plane. Thus, if B and A represent quantities before and after the swingby,

$$\frac{\mu}{2a_{\rm B}} - \frac{\mu}{2a_{\rm A}} = n' \left\langle \sqrt{\mu p_{\rm A}} \cos i_{\rm A} - \sqrt{\mu p_{\rm B}} \cos i_{\rm B} \right\rangle$$

where p stands for the semi-latus rectum of the orbit  $p = a(1 - e^2)$ .

An example of the usefulness of these relationships is the calculation of the orbital eccentricity after a swingby that provides a certain energy change. Suppose it is desired to change the orbit period from 10 days (the period of the Earth-Moon transfer orbit) to 40 days ( the outer orbit of the  $[1,1,1]^*$  DLS). Suppose, further, that the motion takes place entirely in the Earth-Moon plane ( $i_A = i_B = 0$ ). Because the angular momentum of the transfer orbit is known ( $[\mu p_B]1/2$ ), it is easy to find the value of post swingby angular momentum. From this, one can get the eccentricity after the swingby and then, for example, calculate the time required to go from the swingby to apogee of the outer orbit.

The use of the Jacobian integral is of great value in evaluating practical aspects of the launch strategy such as the selection of the translunar trajectory. The value of the Jacobian constant, C, for a 100 n. mi. by 60.2 Re translunar trajectory is  $C_T = -1.210 \text{ km}^2/\text{s}^2$  if the orbit is in the Earth-Moon plane. But the Jacobian constant for the [1,1,1] DLS orbit is  $C_{[1,1,1]} = -1.121 \text{ km}^2/\text{s}^2$  and this orbit must be in the Earth-Moon plane to ensure a second lunar swingby after apogee of the outer orbit. If, however, the transfer orbit is highly inclined, the value of its Jacobian constant is much closer to that of the [1,1,1] orbit. If the transfer orbit is inclined 52.5° to the Earth-Moon plane, the value of C is  $C_{T52.5} = -1.136 \text{ km}^2/\text{s}^2$ , a value much closer to the [1,1,1] orbit than before.

The considerations above indicate that, if it is desired to go directly from the transfer orbit to the [1,1,1] DLS orbit, it would be better to use a high inclination transfer orbit. If, on the other hand, there is some reason why a high inclination transfer is not practical, it would be better to target for the [2,3,1] DLS orbit, whose constant is  $C[2,3,1] = -1.214 \text{ km}^2/\text{s}^2$ . If the values of C are not the same for the transfer orbit and the DLS orbit, the difference in "energy" will have to be provided by the spacecraft propulsion system if the desired orbit is to be achieved.

The value of C can be efficiently changed at perigee or apogee of an orbit by the application of a small propulsive maneuver. The amount of change per unit  $\Delta v$  can be estimated by noting that

 $C = v^2/2 - \mu/r - n'rv\cos i$ 

where v and r represent the speed and radius at either perigee or apogee. Differentiating with respect to v gives

$$\Delta C = \frac{\partial C}{\partial v_A} \Delta v_A = \langle v_A - n'r_A \rangle \Delta v_A$$

<sup>\*</sup> This shorthand notation for classifying the DLS orbits is described in the original paper Ref. 1.

and similarly for perigee. The  $\Delta v$  required to correct  $\Delta C$  at the 140.9 Re apogee of the [1,1,1] orbit, for example, is just  $\Delta v_A = \Delta C/(-2.07)$  km/s. Thus, the correction required for the difference between an in-plane transfer orbit and the [1,1,1] DLS would be about 43 m/s if done at apogee of the [1,1,1] outer orbit whereas a 52.5° out-of -plane transfer orbit would require only about 7 m/s.

The analytical considerations above contain no mention of the dynamics of the actual lunar encounter. As the methods for analyzing this phase of the mission are well documented, they will be presented very briefly to clarify the nomenclature and the reader is referred to the literature for further details.

#### The Dynamics of Moon-Passage

Fig. 1 is a velocity diagram of the relavent quantities in a typical lunar swingby showing the velocities in an Earth- relative frame and their analogs in a Moon-centered coordinate system. If  $V_B$  is the Earth-relative velocity vector on the transfer at lunar encounter (before the swingby), then

$$\mathbf{V}_{\infty B} = \mathbf{V}_B - \mathbf{V}_{M'}$$

is the incoming hyperbolic excess velocity of the Moon passage hyperbola and  $V_M$  is the Moon's velocity vector. A similar equation represents the transformation back to Earth-relative velocities after the swingby, that is,

$$\mathbf{V}_A = \mathbf{V}_M + \mathbf{V}_{\infty A}.$$



Fig. 1 Zero-Patched Conic Velocity Diagram

The key to the zero-patched conic method is to require

$$|v_{\infty_A}| = |v_{\infty_B}|$$

and then to treat the swingby as a two-body problem during Moon passage.

The required angle,  $\alpha$ , between the incoming and outgoing excess vectors can be obtained from the vector relationships above. The eccentricity of the hyperbola is then found from  $\arcsin(1/e) = \alpha/2$ , and the radius of perilune passage is obtained as  $r_p = \mu(e-1)/v_{\infty}^2$ . These methods have been applied to the swingby from a highly inclined Earth-to-Moon transfer orbit to the [1,1,1] DLS orbit. The perilune radius of the swingby was about 4100 km, well above the 1738 km radius of the Moon.

#### Numerical Verification

The swingby distance calculated above is actually somewhat conservative as it is based on the assumption that the Moon is in a circular orbit. In June of 1992, the geometry is such as to favor encounters near the lunar perigee. In order to obtain a preliminary idea of these effects, and to verify, in a general way, the conic approximations outlined above, a numerically integrated transfer to the [1,1,1] DLS was carried out. The simulation includes the effects of Earth oblateness, and the gravitational effects of the Sun and the Moon on the entire trajectory. The launch time was varied until the post swingby Earth orbit had approximately the same energy as the outer orbit of the [1,1,1] DLS orbit. Because the encounter occured near lunar perigee, the swingby was considerably more powerful than the ones analyzed above. The swingby radius, instead of (the circular lunar orbit equivalent) 4900 km, was about 8800 km leaving plenty of margin to target to other, larger, outer orbits if desired.



Fig. 2 Integrated [1,1,1] DLS Trajectory

The aim point of the first swingby was further refined until a second lunar encounter was achieved on the inbound leg of the outer orbit. The orbit after the (28000 km) second swingby had dimensions 4.2x89 R<sub>e</sub>, slightly more elliptic than the inner orbit obtained from the conic approximations but, nevertheless, close enough to ensure a third encounter with the application of modest propulsive maneuvers applied near perigee and apogee of the orbits achieved.

Fig. 2 shows an ecliptic plane projection of the numerically integrated DLS orbit including the transfer trajectory from Earth launch to the first lunar swingby. Mean orbital elements were used for the ephemeris and the integration was carried out using a Cowell formulation and a seventh-order, ten-cycle Runge-Kutta integration scheme. The integration revealed shadows on the transfer orbit of about 11 minutes after launch and about 40 minutes prior to the first lunar swingby. The inner orbit of the DLS orbit was also found to enter Earth shadow for about 45 minutes near perigee passage. The orbit is shown in a non-rotating frame.

#### **THE EARTH-MOON SYSTEM IN 1992**

Fig. 3a and 3b show the Earth-Moon-Sun geometry in late June of 1992. The figures show that the ascending node of the Moon's orbit on the ecliptic is close to 90° west of the vernal equinox. This means that the inclination of the Earth-Moon plane is about 24° or about midway between the maximum and minimum values achieved as the Moon's node regresses on the ecliptic once every 18.6 years.



Fig. 3a Morning Launch Geometry

The diagrams also show two 28.5° orbits representing the results of 90° azimuth launches from ETR at two different times of day. These two times are designated morning and evening launches because the apogee of the Earth-Moon transfer orbit will be generally in the direction of the Sun. The launches have been timed to place either the ascending or descending node on the Earth-Moon plane so that either orbit will contain the Moon at the desired arrival time. Now the Earth-to-Moon transfer trajectory can be initiated from either orbit near its node on the Earth-Moon plane opposite the direction to the Moon at encounter (behind the Earth in Figs 3a and 3b).



Fig. 3b Evening Launch Geometry

The transfer orbit from an evening launch will have a low inclination ( $\sim$ 4°) to the Earth-Moon plane while the morning launch will yield a high-inclination transfer (about 50°). Either trajectory will encounter the Moon at the same point in its orbit without the need for a performance penalty on the translunar injection burn. The main difference in the two trajectories is that the high-inclination transfer will yield a higher excess speed with respect to the Moon. This will improve the overall ability of a lunar swingby to change the orbit throughout the mission (see Ref. 2).

The lag angle of about 34° shown on the figure is approximately the amount of angular distance between the Moon and Sun at lunar swingby required for the post-swingby orbit to have its apogee on the Earth-Sun line when the spacecraft arrives at apogee about 16 days after encounter. This corresponds to the option to transfer to the outer orbit of the [1,1,1] DLS orbit via an outbound swingby. In case launch were delayed on the day shown in the figure, the transfer could be retargeted to the inbound swingby point about 100° East of the encounter point shown. Such a launch would occur about 7.6 days (100° of lunar motion) later than the opportunity of the diagram and, of course, there would be two launch windows on that day also. These daily launch windows can be as much as 1/2 hour wide corresponding to about 7.5° of Earth rotation or about 50000 km of distance on the Moon's orbit. Even without the focusing effect of the lunar gravity, the worst miss to be expected is about 25000 km which is well within the Moon's sphere of influence and can be easily removed by a small maneuver near apogee of the transfer orbit.

Thus the Double Lunar Swingby orbit provides two launch opportunities per month and the use of high-inclination transfers permits two launch windows per day even in the situation analyzed here where the target DLS orbit is the [1,1,1] option. It is to be expected that further analysis will reveal even more flexibility as the other transfer options are studied.

#### Swingby Transfer to L1 Halo Orbit

After the scheduled sequence of DLS orbits, the WIND spacecraft will transfer to the L1 Halo injection point. This can be accomplished by retargeting one of the outbound lunar swingbys for a (perturbed) apogee of about  $1.5 \times 10^6$  km and the spacecraft makes the slow transfer to the L1 injection point about  $1.25 \times 10^6$  km from the barycenter of the Earth-Moon system. Optimal Halo orbit transfer trajectories from the DLS orbits tend to be somewhat longer than than those studied for ISEE-3/ICE. Minimum (single) injection impluse transfers require 120 to 140 days and result in injection impulse values less than 50 m/s for the minimum amplitude Halo orbit with dimensions of about 450,000 km in the sunward direction and about  $1.3 \times 10^6$  km in the circumferential direction.



Fig. 4a Swingby Transfer to L1 Halo Orbit

It is to be expected that the use of multi-impulse transfers to the Halo injection point and careful planning to take advantage of the perturbations may yield transfer  $\Delta V$  requirements less than the 50 m/s value quoted above and save propellant for Halo orbit mission stationkeeping. Fig. 4a is the result of a rather crude optimization that shows transfer to the Halo injection point via a lunar swingby compatible with a [1,1,1] DLS orbit.

The reader may wonder why the injection into the Halo orbit in Fig. 4a takes place so far from the Sun-Earth line where one might expect the optimal maneuver to be located. What is not shown above is the out-of-plane component of the transfer orbit. Fig. 4b shows the transfer as seen from the Sun and shows that the post-swingby trajectory is slightly cocked with respect to the ecliptic plane which contains the major axis of the desired halo orbit. This new point of view shows that the lunar swingby should have been targeted to raise the Moon-L<sub>1</sub> portion of the transfer a little closer to the ecliptic plane to make the transfer orbit tangent to the halo orbit along the Sun-Earth line at the Earthward edge of the halo. It should be noted that these trajectories are extremely sensitive to the lunar swingby conditions and require a delicate "touch" on the part of the searcher to prevent escape from the Earth-Moon system or return to the lunar orbit on a trajectory resembling the [1,1,3] DLS orbit.



Fig. 4b Swingby Transfer to L<sub>1</sub> -- View from Sun

#### THE BACK-FLIP: USING THE THIRD DIMENSION

Most of the discussions and diagrams of the previous sections have been two-dimensional in the sense that the motion is presumed to be essentially in or near the ecliptic plane or the Earth-Moon plane which is itself near the ecliptic. But the strength of lunar gravity assist need not be confined to planar acrobatics. It is possible, for example, to target a minimum energy translunar trajectory for a near polar Moon-passage hyperbola that will yield a post-swingby orbit that is inclined nearly 60° to the Earth-Moon plane. Such a maneuver was dubbed orbit cranking in Ref. 3 and is discussed in detail there. The cranking to high inclinations using the Galilean satellites at Jupiter, however, required repeated encounters with the satellites. In the Earth-Moon system, the orbit may be boosted in inclination through the maximum crank angle in a single lunar encounter. This opens up the possibility of a very rapid (14 day) Moon-to-Moon transfer that permits the mission designer to convert a sunward DLS orbit into an anti-sunward DLS simply by adding a 14 day high-inclination segment to the mission profile.

Fig. 5 is a diagram showing the Back-Flip maneuver and is representative of the results of numerical integrations that verify the feasibility of this transfer mode. If the Moon had no effect on the transfer other than the initial swingby, the conditions for the Back-Flip would be that the transfer orbit have exactly the same shape and size as the Moon's orbit. Actually, the lunar gravity acts on the trajectory throughout the transfer and, as the transfer lasts only half an orbit, tends to modify the transfer orbit asymetrically. The inclination of the transfer orbit can be calculated from a knowledge of the Jacobian constant of the preswingby trajectory, C<sub>T</sub>. The idealized inclination of the BackFlip, I, is then given by

$$\cos I = \frac{\left[ -\frac{\mu_{\rm E}}{2a'} - C_{\rm T} \right]}{n' \sqrt{\mu_{\rm E} a' (1 - e'^2)}}$$

where the primes refer to the Moon's orbit and  $\mu_E$  is the Earth's gravitational constant. The value of I used to start the search for a numerically integrated transfer was found to be 42.5° with respect to the Earth-Moon plane.



Fig. 5 The Back-Flip: 180° Moon-to-Moon Transfer

#### CONCLUSIONS

The basic analytic techniques for lunar swingby missions were outlined and used in some preliminary studies of the ISTP cislunar missions. The use of targeting to either DLS lunar encounter permits two launch opportunities each month and the use of high-inclination transfers provides twice-daily launch windows during those opportunities. A discussion of the geometry of the Earth-Moon system during the desired launch month was included and a low impulse transfer to an L<sub>1</sub> halo orbit was identified. A high-inclination, near circular Moon-to-Moon transfer mode was identified that allows rapid transfer from sunward to anti-sunward DLS orbits. The great flexibility provided by the Double Lunar Swingby orbit was cited as a distinct advantage in mission and mission operations planning that will have major implications for the ability of the mission profile to meet the scientific goals of the ISTP program. The artistic elements of gravity-assisted mission design were contrasted with the analytic aspects and it was suggested that more careful attention to the esthetics may be important to creating an academic environment conducive to the discovery of new concepts.

### References

1. Farquhar, R.W., and Dunham, D.W., "A New Trajectory Concept for Exploring the Earth's Geomagnetic Tail," *Journal of Guidance and Control*, Vol. 4, No. 2, March-April 1981, pp. 192-196.

2. Egorov, V.A., "Three-Dimensional Lunar Trajectories," NASA TTF-504, 1969, pp 104-144.

3. Uphoff, C., Roberts, P.H., and Friedman, L.D., "Orbit Design Concepts for Jupiter Orbiter Missions" *Journal of Spacecraft and Rockets*, Vol. 13, June 1976, pp. 348-355.

4. Niehoff, J., "Gravity-Assisted Trajectories to Solar System Targets," *Journal of Spacecraft and Rockets*, Vol. 3, No. 9, Sept. 1966, pp. 1351-1356.