# Introduction

This book is intended to teach some of what I know about rockets. The specific technical details are quite different for amateur rocketry than they are for professional "big" rockets. In many ways, the amateur has more formidable obstacles than the professional. The amateur must deal with drag as a constant problem where the professional can expect to lose only about 200 meters per second (m/s) to drag on a typical launch to low Earth orbit. There, the total change in velocity ($\Delta V$) is about 9200 m/s.

The amateur, on the other hand, is restricted to an *I*sp in the range of 80 to 90 seconds and must suffer the slings and arrows of stage propellant mass fractions under 0.7. The outstanding safety record of amateur rocketry is one of the reasons for the low *I*sp and mass fraction of amateur rockets. Amateur Rocketry, like Amateur Radio, is an education in itself; it teaches the amateur how to get the right answer.

In one way or another, I have spent most of my professional life in the rocket/space-exploration industry. Although I am not a rocket engineer, I have done many mission designs which require that I combine existing stages or propulsion systems, in various ways and under multiple constraints, so as to obtain the best performance. The computer program that comes with this book is designed to assist in that very task.

The substance of this book is an exposition of some very simple things I have learned from books, from experts, and from experiments. The computer program described here is one I have used for many years with great value to my own understanding and to my space mission designs. I hope that the reader will benefit from the things I have learned in consideration of the examples and solutions discussed in this book.

Chauncey Uphoff
Niwot, Colorado
1993 December 30

## Chapter I

## An Interest in Rockets

The purpose of this chapter is to explain that the rocket equation is identical in form to the interest equation, the equation to which so many of us pay dearly at the beginning of each month.

The rocket equation (or the interest equation) will fool you almost every time. Just when your think you know what will happen, the amount you have to pay out will be a lot more than you thought. "No," you say, "I pay the same amount each month. " That's because the money industry understands the interest equation; they spread out your payments over a long time, so you won't see ("feel" is a better word) how much you're really paying. This is an idea that is familiar to most people who have credit cards or bank accounts. The use of money has value; when you borrow money, you should expect to pay a percentage for using the money. When you loan money (or open a savings account), you should expect to get a percentage of the amount as "interest" or payment for the use of the money.

The rocket equation is the same way, except that very few politicians are interested in the savings in propellant or the launch funding required to go to low Earth orbit. When more of their constituents make their livings from space transport, they will pay more attention to the rocket equation.

What makes the rocket equation the same as the interest equation? The answer is that the rate of change of something, depends upon the amount of the something there is. Suppose you invest $100 at an annual interest rate of 5%. This means that you let someone else use your money for a percentage of the amount you loan or invest. At the end of one year, the bank (or the person to whom you loaned the money) owes you $105.

At the end of the second year, the bank owes you 1.05 times $105 or $110.25. At the end of the third year, it's $115.76. At the end of the 10th year the bank owes you $162.89. After 20 years, the value of the loan is $265.33 and, after 30 years, you have $432.19. This kind of growth is surprising to many people when they first try to understand it. During the first few years, it appears that you are getting only about $5 per year for the use of your money. In thirty years you would expect to make about 30* $5 or about $150 so that you would expect to get back about $250 for your initial investment.

But you get back nearly twice that much - what happened? What happened was the extra 25 cents after the second year that you thought was unimportant. That extra quarter represents the exponential growth of the future value of the loan. So what does this little lecture on saving your money have to do with rockets? Be patient. Let's double the interest rate and see what happens.

Start with the initial $100 but now suppose you find a mutual fund that will pay 10% annual percentage rate (APR) on your money. Somewhere in the back of your mind, you expect to get back about twice as much as you would with the 5% bank loan. So, at the end of the first year, you would have $110, just as you expected. At the end of the second year you would have $121. Wait a minute; the extra quarter from before is now a dollar. But you only got twice the interest rate. Now watch what happens as time increases. After 5 years you have $100 x (1.10) x (1.10) x (1.10) x (1.10) x (1.10) = $161.05 or, in mathematical language, the future value of the loan, F, in terms of the present value of the loan, P, is

$$F = P \times (1+i)^{n},$$

where $i$ is the interest rate expressed as a number, not as a percentage, and n is the number of years during which the interest is compounded (or accumulated) in the bank or mutual fund. Let's see what happens to your $100 after 10 years at 10%.

$$F = \$100\text{x}(1.10)^{10} = \$100\text{x}2.59374 = \$259.37.$$

After 20 years, the value of the loan is $672.75 and, after 30 years, it's a surprising $1744.94. If you guessed what you'd have after 30 years, (at about $10 per year) you'd have said about $400, maybe a little more. But you end up with over 4 times your guess based on a linear extrapolation of 1 year's interest. OK, now guess what your $100 would be worth to your grandchildren in 90 years at the 10% interest rate. Guess first and then work it out. Then work out how much the interest on a $100 loan would be for 40 years, the average working lifetime of most people, at 18%, the average percentage rate charged by most of the credit card companies. Try to guess first, and then calculate F = $\$100\text{x}(1.18)^{40}$ or $100*(1.18)x(1.18)x(1.18)x ... 40 times.

The answer is an astounding \$75,037.83. Little wonder the credit card people encourage you to borrow more money. Of course, you don't pay this much because you are not permitted to borrow the money for 40 years. If you pay the interest every year, the total amount for borrowing is only \$720 or \$18 per year. But if you let the debt ride for 40 years, you'll pay \$1875.95 per year on a \$100 loan.

The rocket equation behaves the same way as the interest equation, except that the 40 or 90 years are compressed into a few seconds of propellant burntime. That's why it's important to understand how easily you can be fooled by the human tendency to extrapolate linearly. One of the best things you can learn in economics or in the physics of rockets is to know when to guess and when to do the calculations.

The title of this chapter has a double meaning. The interest equation is just an everyday example of exponential growth. But your interest in the subject of rockets will be the most valuable thing you can have to help overcome any nagging doubts that you can really understand this stuff. Don't be afraid of the equations; they're just a shorthand way of writing down what is going on. A strong interest, in any subject, will overcome almost any lack of initial expertise.

The rest of this book is a series of chapters of increasing complexity and detail, designed to bring the novice as far as he or she wants to go toward an understanding of the optimal staging problem. The student is cautioned that some of these developments require knowledge of calculus. The proper use of the program that goes with this book, however, requires only some common sense, and an understanding of the principles behind the terms "stage propellant mass fraction,' and "specific impulse." As these terms are not always used in amateur rocketry publications, they are described in some detail, and in terms of familiar quantities, prior to the section on multi-stage optimization.

The reader is reminded that you don't have to understand all this stuff to enjoy amateur rocketry. But if you want to design your own high-performance stages, or work in the professional rocket industry someday, the material of this booklet and the use of the program Impulse® will go a long way toward helping you achieve those goals.

Chapter 2

## $\Delta$V Over C, That's the Key

Now let's look at the simplest kind of rocket problem there is. We're going to ignore drag; we're going to ignore structural mass; we're even going to ignore gravity! We start with a perfect rocket in deep space, far from any gravitational fields, and with no air to slow us down. The rocket is perfect in the sense that it ejects its exhaust mass directly opposite the velocity vector, and it ejects mass at a perfectly constant speed and flow rate. The speed of ejection, or exhaust speed will be called c, (expressed here in m/s), and the mass flow rate will be called *mdot* and will be expressed in kilograms/sec.

One of the simplest results of Isaac Newton's work is that a body in motion will continue with that motion unless acted upon by some outside force. What Professor Newton meant by "motion" is what we now call momentum, the product of mass and velocity. Thus, if we have a rocket, propelled by its own internally stored energy, with no forces acting from out side the original rocket and its propellant, the center of mass of the rocket will continue moving exactly as it would have if no propellant were burned. This gives us a powerful tool for predicting the motion of rockets without (necessarily) having to resort to the differential calculus. Let's see what we can get from Newton's First Law of Motion without calculus.

Let the initial mass of the rocket, $m_r$, and its propellant, $m_p$, be called $m_0$, so that $m_0 = m_r + m_p$. Now, let the propellant be ejected from the rocket in a direction opposite the rocket's initial velocity vector, $\boldsymbol{V_0}$. The propellant, which you can think of as a brick thrown out the back of the rocket by a little gnome whose job is to throw propellant bricks at a certain speed, leaves the rocket at the speed, *c*, which is fastidiously controlled by the gnome's ability to throw bricks. Don't worry about how much the gnome weighs, or how much food he requires; it doesn't matter for now.

Assuming that the rocket is traveling in a straight line, and the gnome is a straight shooter, we can ignore all directions except the original direction of motion. To conserve momentum, then, we must have, after the gnome throws our a brick of mass $m_p$, at speed, *c*, relative to the rocket,

$$m_0V_0 = (m_r + m_p)V_0 = m_r(V_0 + \Delta V) + m_p (V_0 - c)$$

Now don't panic if you're not mathematically inclined; this is just the mathematical way of saying that the original mass times the original speed of the rocket is the same as the mass of the rocket times the speed of the rocket plus the mass of the brick times the speed of the brick, after the propellant brick is ejected from the rocket. Fig. 1 shows the situation. Just remember that you're adding up the momentum of the pieces of the original rocket before and after the gnome throws the brick out the back. Professor Newton has a hold on the gnome; the gnome, and the rocket he's riding in, must always gain enough momentum from the brick throwing, to just cancel the momentum of the brick that was thrown out.

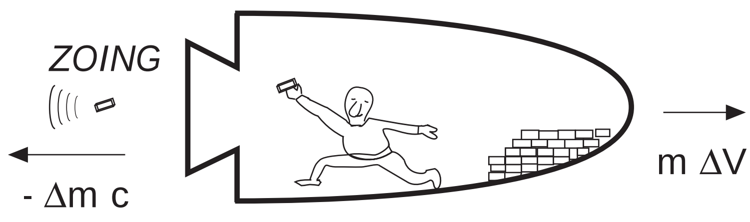


**Fig. 1 Rocket, Gnome and Bricks**

Eliminating the identical terms in the equation above gives us

$$m_r\Delta V = m_pc\,, \quad \text{or,} \quad m_p/m_r = \Delta V/c.$$

Now this is a simple equation. Can all this rocket business be this simple? No, there are some very complicated things involved in the aerodynamics of rockets, but, if you eliminate the effects of turbulence, friction drag, and all the other aerodynamic forces on rockets, the simple equation above is practically all there is. Of course, there is no cooperative gnome to throw bricks in a perfectly straight line out the back of the rocket, and the real gnome (the chemical energy stored in the atoms of the propellant) probably can't maintain a constant flow of bricks, but our current technology can provide us with a pretty cooperative gnome. The Thiokol Star 37XFP, one of the best solid rockets available commercially, provides a nearly constant thrust of 35,000 newtons, and a nearly constant specific impulse of 290 seconds throughout its 66 second burntime. The stage propellant mass fraction of the Star 37XFP is an astounding 0.924. In deep, or close Earth orbital space, the effects of turbulence and friction drag are negligible. There, we can use the simple rocket equation, including the effects of inert (structural) mass, and accounting for nose drag, to design missions that use a minimum of propellant for a fixed payload, or missions that provide a maximum payload for a minimum initial mass.

Now, returning to the rocket equation, we find that we're going to need some of that dreaded subject, calculus. But first, let's get the correct relationship between $m_p$ and $m_r$ for use with the equation above which we derived from the condition that momentum must be conserved. Let's call one of the bricks $\Delta m$ and call the mass of the rocket m, dropping the subscripts.

In the language of the calculus, we write the momentum conservation equation for a little chunk of $m_p$, which we are now calling ($-\Delta m$). This equation is just

$$(-\Delta m)/m = \Delta V/c.$$

Wait a minute, how come we substituted $-\Delta m$ for $m_p$ in the equation from the previous page? The reason is that we are now keeping track of the mass of the rocket, not the mass of the ejected propellant. Thus, because the mass of the rocket and the all the exhaust must always add up to the initial mass, the relationship between a change in $m_p$ and $\Delta m$, in the momentum equation, is negative. This may seem a little tricky, but don't worry about it. You can always figure out which way the rocket equation works, because it only makes sense one way; a decrease in mass of the rocket, corresponds to an increase in the $\Delta V$ added. In the words of an anonymous friend from the past, " There are two things you can't do in astrodynamics; you can't push a rope, and you can't throw a rocket into suck." The anonymous friend must not have heard of the interest equation.

So far, the gnome has been throwing out discrete, measurable bricks. As the bricks become smaller and smaller, the equation for conservation of momentum becomes what mathematicians call a differential equation - another phrase to throw fear into the hearts of the uninitiated. Don't worry about it; it all makes good common sense. Physics is simple, if you don't let it scare you.

If you want to alienate a mathematician, tell him that the only difference between a discrete equation and a differential equation is to change the symbols from $\Delta$'s to $d$'s. He will probably scream and yell, and tell you that you have no understanding of the concept of a limit - the worst insult to a mathematician. The idea of a limit is as important to a good engineer as it is to a good mathematician, although probably neither would agree.

Here's an example of a limit that will make good practical sense. Suppose you try to figure out how much $\Delta V$ (change in speed for our simple rocket) you'll get if the gnome throws out a 10 kilogram brick, from a 100 kilogram rocket, at an exhaust speed of 1000 meters/sec. The momentum equation says you'll get, after the gnome throws the brick,

$10/90 = \Delta V/1000$, which implies that $\Delta V = 111.1111$ m/s.

But what happens if the gnome throws out two, 5 kilogram bricks, instead of one 10 kilogram brick? Try it.

$5/95 = \Delta V_1 / 1000$ implies $\Delta V_1 = 52.6316$ m/s.

Now, throw off another 5 kg brick at 1000 m/s relative to the rocket.

$5/90 = \Delta V_2/1000$ implies $\Delta V_2 = 55.5556$.

The total $\Delta V$ achieved by the expulsion of the two, 5-kilogram bricks, each with a rocket-relative speed of 1000 m/s, gave the rocket a $\Delta V$ of 108.18716 m/s. Wait a minute - that's not the same as throwing off a single 10 kilogram brick. What happened to conservation of momentum? Good question. Check it out . . .

Initial momentum = 100 kg x 0 m/s = 0 kg m/s
After first brick: Momentum = 95x(52.6316) + (5)x (-1000). = 0

Now throw the second brick:

Momentum = 90x(55.5556) + 5x(-1000) = 0.

Now, add up all the momenta of the three particles after the throwing of the second brick: The rocket is going to the right at a rate of 108.18716 m/s, the first (5 kg) brick is going to the left at 1000 m/s, and the second (5 kg) brick is going to the left at a rate of 947.3684 m/s. Add up the products of the masses and the speeds; you'll get zero, the initial momentum of the rocket at liftoff. So, there's no problem with the total momentum; it's just a weird thing that the speed of the rocket depends slightly on how many little chunks of the initial mass you choose to throw out, even though the speed of ejection and the total amount of mass ejected is the same.

It's important to understand this; someday you might be asked to design a pulse rocket like the Orion, and you'll need to understand the difference between discrete expulsions of exhaust mass and a continuous flow of very small particles. But, for now you can assume that the propellant flows out of the rocket at a constant rate and that every little bit of propellant ejected adds to the speed of the rocket.

Try the last experiment of cutting down the mass of the bricks, again and again, until you're sick of doing it. You'll get very bored because, after the first 6 or 8 times you cut the mass in half, you'll find out that you get the same answer for the ΔV. The whole process will have you asking why you listened to the author. This is the essence of boredom; no matter how small you make the Δm, you always get the same answer. What fun is this? I want to launch rockets.

If you followed the process of cutting the mass of the expelled propellant in half until you were very bored, you understand the concept of a limit. This is a very good thing to learn however frustrating to the impatience of youth, because it will help you learn to design rockets and many other useful things.

The limit of the process described above is

$$- \Delta V/c = \ln [m/m_0] ,$$

where ln means the natural logarithm and, for the example given above, gives $\Delta V$ = 105.3605 m/s, the same number you got by chopping the propellant bricks into smaller and smaller chunks.

We can write the rocket equation in the following form,

$$m_0 = m\, e^{(\Delta v/c)},$$

where $m$ is the mass of the rocket, $m_0$ is the initial mass of the rocket, $\Delta V$ is the amount of speed imparted to the rocket, and $c$ is the exhaust speed of the propellant with respect to the rocket. We can think of this as the liftoff mass required to deliver a certain payload mass (m) to a certain $\Delta V$ using propellant with an exhaust speed $c$. In the equation above,

$$e = 2.718281828 \ldots\ldots \;.$$

This is a number, like π, that goes on forever without repeating itself. This is a very important number, studied by most of our best mathematicians, that crops up in all kinds of natural processes.

Now rewrite the rocket equation as

$$m_0 = m\,(1\ + 1.718281828\ldots)^{(\Delta V/c)},$$

and we see that the rocket equation is like a loan at 171.828% interest, with a compounding "period" of $\Delta V/c$. This quantity plays the role of the number of years the "money", m, is loaned or borrowed and $m_0$ is the "future value" of the loan. Thus Nature is a very tough moneylender (see Fig. 2), even with the perfect rockets we have assumed above. In the next chapter, we shall see the effects of structural or "inert" mass that must be carried along, as excess baggage, in all real rockets.

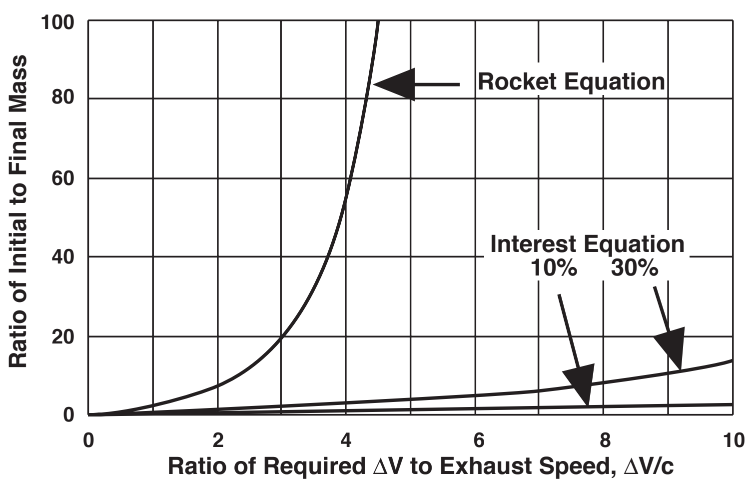


**Fig. 2** ***Initial mass required to deliver 1 kg to different values of $\Delta V/c$. $\Delta V/c$ plays the role of the number of years of a loan at 178.3% per year. $m_0$ is like the future value of a loan of the amount m.***

Chapter 3

## You Can't Push a Rope

The careful reader may have asked why the $\Delta V$ acquired by the rocket doesn't depend on the thrust of the rocket. Everybody knows the higher the thrust, the more oomph you'll get out of the rocket. That's true, but what do we mean by oomph? Usually we mean speed, or change in speed, or how fast the coyote is moving when he slams into the solid rock wall. But that "definition" comes from our Earthbound experience where forces have to overcome other forces. Out in deep space, it doesn't matter a hoot how much thrust is applied; the change in velocity will still be $c\ln(m_0/m)$ because momentum must be conserved. In real life, of course, too much thrust would crush the skin of the rocket and mush the gnome into flat goo on a forward bulkhead of the vehicle. Why the forward bulkhead? Because the gnome is throwing the bricks, and, as long as the vehicle gets acceleration from the brick-throwing, the reaction force must be transmitted to the rocket through some structure (like a bulkhead) that is connected to the gnome.

Well, now we're really into it. The paragraph above has about five words and as many disciplines so far not encountered by the amateur rocketeer. There was oomph - which we'll call acceleration; there was the word mushed, let's call that strain; there was the word hoot, which is a quantity less than the smallest amount you can imagine; and there was the word structure, a word which has many meanings. In rocketry, structure usually means you're going to have to give up some performance you thought you had in your original design.

When I was at JPL, one of my colleagues, J.R. French, used to give me the business for certain kinds of neglect. One of his pet peeves was my lack of explicitness whenever I used the word fuel when I meant propellant, a combination of fuel and oxidizer. Another is that I would come to him with a design for his approval that required a spacecraft and rocket motor structure made of what he called Impervium/Unobtainium alloy. Impervium is a material that has infinite rigidity, and no mass. Unobtainium is any material that contains Impervium.

Jim French is one of the best rocket engineers in the Western world. I learned very quickly never to discount anything Mr. French told me about rockets. The most important thing (about rockets) I learned from him is NOT to underestimate the structural factors. Every new rocket design is different in some way and, almost certainly, the stage propellant mass fraction for any stage will be overestimated by an amateur like the author.

The stage propellant mass fraction is a very important factor in rocket vehicle design. It is the ratio of (propellant) to (propellant plus everything else) in a stage of a rocket. It is a term that is much more descriptive than many others used like " structural factor," "tankage factor," or usually, "mass fraction." In this book, and in the operation of the computer program described later, the term "stage propellant mass fraction" shall be designated $\lambda'$, and means

$$\lambda' = m_p / (m_p + m_s) ,$$

where $m_p$ is the mass of the useful propellant and $m_s$ is the mass of the structure, any unburned propellant, and any ham sandwiches left in the rocket stage structure by workers or sightseers. The ham-sandwich factor is very small for professional rockets. In amateur work, you might look for small pieces of glue, tape, or a very small part of a ham sandwich. The point is that $\lambda'$ is (propellant mass / stage liftoff mass) for that stage. $\lambda'$ has nothing to do with the mass or characteristics of any stage above or below it in the rocket stack.

The inert mass of a stage, including any structural mass required to attach it to, and later separate it from, an upper stage, is extra baggage for the lower stage(s). For values of $\Delta V/c$ greater than 1, or for low stage propellant mass fractions ($\lambda' < 0.8$), the problem of getting rid of the excess baggage or tankage of the lower stages becomes critical to achieving most mission objectives.

Any extra (inert) mass from previous stages should be dropped if the rocketeer wants maximum performance. Your rocket will go away from any such chunks of mass " as if they were tied to a tree."

There is one more point, not always mentioned in rocket catalogs, that should be identified before we optimize multiple stages to gain a desired $\Delta V$. That is the dreaded concept of SPECIFIC IMPULSE.

Specific impulse is just the total impulse divided by the propellant mass. It is a way of describing how much impulse the rocket gets out of each kilogram of propellant. So what is impulse?

Impulse is the product of force and the time during which the force acts. It has the units of momentum, (mass times velocity) the stuff we conserved to derive the rocket equation in chapter 2. We'll discuss this more in the next chapter, but, for now, we need only notice that the rocket seems to have a force acting on it; its momentum changes. When we conserved momentum before, it was for the rocket *plus* the exhaust. Each time the gnome throws a brick, there is a reaction force, called the momentum thrust, on the rocket. It turns out that this force is equal to *mdot* times *c,* the product of mass flow rate and the exhaust speed. Assuming that the thrust is constant, we get, for the total impulse, $I_T$,

$$I_T = F \times \Delta t = mdot \times c \times \Delta t,$$

where $\Delta t$ is the burntime of the motor. Notice that the units are newton seconds, the same thing as kg m/s$^2$ times seconds. This is the same as kg m/s, the units of mass times velocity. But specific impulse is the total impulse divided by the propellant mass, so the units are the same as those of velocity or speed, m/s. Using the equation above and noticing that $mdot = m_p/\Delta t$, we get

$$I\mathrm{sp} = I_T/\mathrm{m_p} = (m_p/\Delta t \times c \times \Delta t) / m_p = c.$$

***What? You mean the specific impulse is just the exhaust speed?*** Well, yes and no. In the mks (meter-kilogram-second) system of units we've been using, the answer is yes. In common practice, where many people use a modified English system with Lbm (pounds mass) as the unit of mass instead of slugs, the units of Isp are Lbf (pounds force) times seconds divided by Lbm. So if you take the average thrust in Lbf, multiply it by the burntime in seconds and then divide by the mass of the propellant in Lbm, you get a number that has the units of seconds although it is properly quoted as Lbf sec / Lbm. This has become such standard practice that specific impulse is nearly always quoted in seconds. The relationship between Isp and the exhaust speed, c, is then , $c = gI\mathrm{sp}$ ,where g is the acceleration due to gravity at Earth's surface.

***Henceforth, in this book, and in the inputs for the Impulse® program, Isp will be quoted in seconds, no matter which units we're using. The exhaust speed will always be given by the product of g and Isp.***

Most of the Estes rocket motors have an *I*sp of about 82 seconds which means that the exhaust speed is about 804 m/s or about 2638 ft/s. If you take the total impulse quoted in newton seconds, and divide by the propellant mass in kilograms (g/1000), you'll get the exhaust speed in m/s. Then divide by $g$ = 9.8066 m/s$^2$ to get *I*sp. Then convert back to the English system if you wish. If you are given the total impulse in Lbf seconds and the propellant mass in Lbm, just divide $I_T$ by $m_p$ to get the *I*sp in seconds. Then multiply by $g$ = 9.8066 m/s$^2$ or 32.1739 ft/s$^2$ to get $c$ in m/s or ft/s respectively.

The specific impulse of a rocket is simply another way of describing its exhaust speed, which happens to be a measure of the impulse per unit propellant mass. *I*sp is just $c$ (the exhaust speed of our propellant) divided by the acceleration due to gravity at the Earth's surface. That is,

$c = g\, I\text{sp}$, where $g$ is 9.8066 m/s$^2$ and $c$ is measured in m/s.

That's all.

Chapter 4

## What's in a Gnome?

In the previous chapter, we started to talk about thrust, burntime, total impulse, and other things that make sense to amateur rocket buffs. But they all canceled out of the equations and we were left with the same old thing we started with, the exhaust speed of the rocket. The reason for having the discussions in this order is that the stage propellant mass fraction, and the exhaust speed (or *I*sp) are the only quantities we need to do a pretty good design for an optimally staged multi-stage rocket.

Of course, if you choose a 1st stage that doesn't have enough thrust to lift the stack off the pad, you've just optimized yourself out of existence. In deep space, it wouldn't matter so much, but on the pad, while they don't affect the staging parameters much, you'll have to pay attention to other things, like the other forces acting on the rocket. Gravity requires that we have a big enough engine to get the stack off the pad. In amateur rocketry, we rely on fins to stabilize the motion, so we need more than minimal thrust to make sure the rocket has enough speed, when it leaves the launching rod, to keep its center of pressure (c.p.) aft (behind) the center of gravity (c.g.) for all reasonable variations of the rocket centerline from its initial trajectory (these are variations in angle of attack).

We will not consider amateur rocket stability in this book, unless it affects the optimizations done later. Estes's TR -1,2,6, and 11 have plenty of good information on how to build and stabilize your models. Where stability will come into play in this book, and in your own designs, is in the additional mass required for fins on a booster stage made up of, say, three clustered D12-0 Estes stages. The extra mass of these stages will move the c.g. far aft on the vehicle and will require some bigger fins on the booster. The mass of these fins will change the overall $\lambda'$ of the booster stage because the stage liftoff mass will be greater than the sum of the three rocket motors while the stage propellant will be the same as the propellant in three motors.

Another consideration will be the mass of the interstage adaptors, usually tape and corks in amateur stages, and any aerodynamic cowling or drag reduction mass required to keep the lower stages from separating prematurely.

Finally, we will have to be concerned with the effects of atmospheric drag, a constant and limiting problem for the amateur. Someday, perhaps not too far in the future, amateur rocketeers will launch small payloads into Earth orbit. Then they can forget about drag after they reach about 30 km (100,000 ft) in altitude. For the present, however, we must assume that atmospheric drag will remain a major factor for amateur designers.

In the following paragraphs, we shall consider the forces, structural mass requirements, and many of the things we attributed to the gnome in the previous discussions. Don't be discouraged if you don't understand all this; you can still learn a lot from using the program. You'll get the most from trying to understand the next few chapters. Then, you can read some other books from the bibliography and then you can start to design your own rockets. You'll make a lot of mistakes that will result in fizzles or what pilots call a "crappy landing. " That's OK.

What is not OK is if you try something stupid like trying to launch a rocket without following the 14 points of the National Association of Rocketry safety code or the recommendations of the manufacturers of the rocket engines you are using. As far as amateur rocket companies are concerned, if you get hurt, they get hurt. If professional rockets have a problem, the government, or the insurance companies will pay. Somehow, most people think that's OK. No it isn't. We all pay, through increased taxes or increased insurance premiums. The more the insurance companies are controlled by the government, the more of that liability will be transferred from voluntary premiums to involuntary taxes.

So enjoy the knowledge you will get from designing optimally configured rocket stages and clusters. But don't blame me if you burn a hole in your foot. Don't be afraid, but don't be careless.

Now, let's see what's going on inside the rocket; we know Estes doesn't put a little super gnome inside each rocket motor. As we saw, earlier, the thrust seems to have nothing to do with the speed of the exhaust. This makes sense; we could have the gnome throwing out very small bricks of propellant only once an hour, or once a day - even once a year, at a very high speed and have virtually no thrust. Indeed, ion engines used for stationkeeping for geosynchronous satellites have Isp's of 1000 to 3000 seconds (exhaust speeds of 10 to 30 km/s! ) but the thrust is measured in millinewtons. Obviously, the thrust level has something to do with how fast the mass is changing, not just with the speed of each individual brick.

Of course it does; that's exactly what Professor Newton said. The Force is equal to the time rate of change of momentum or, for a system with constant mass, F = ma, where a is the acceleration. But for a rocket getting its thrust only from the expulsion of its exhaust mass, the Force (thrust) acting on the rocket is the time rate of change of the momentum. For our perfect rocket, the only force acting on the rocket is the reaction force of the expulsion of the exhaust mass. The total momentum transferred out of the rocket during a burn of propellant $m_p$ is just the mass of the propellant times the exhaust speed, or $m_p \times c$. The thrust obviously depends on how fast that burn is completed.

The rate at which momentum is transferred from the escaping propellant to the rocket is $(m_p \times c) \ / \ \Delta t$ , where $\Delta t$ is the burntime for the amount of propellant $m_p$. But recall that $m_p$ is just the change in the mass of the rocket $(-\Delta m)$ during the burntime $\Delta t$. So, we have

$F = (-\Delta m / \Delta t) \times c = mdot \times c$ as we stated in the last chapter, and the direction of the force is in the direction opposite the exhaust. You can check this out for yourself from the specs for any given rocket motor. Take the mass of the propellant (in kilograms), divide by the burntime (in seconds), and multiply by the (effective) exhaust speed ($gI$sp in m/s). You'll get the average thrust of the engine in newtons. If $I$sp is not given in the engine specs, take the total Impulse, $I_T$ (in newton seconds) and divide by the burntime. This will give the same (average) value of thrust. The Estes D12-0 engine, for example, has an $I$sp of 81.807 sec , a propellant mass of 24.93 g, and a burntime of 1.73 seconds. The calculated average thrust is 11.76 newtons. The "12" in the 12D-0 nomenclature comes from using the closest integral number of newtons. Actually, the thrust of the Estes "D" series motors peaks at about 28 newtons, but the burntime average is near 12 newtons. Professional (solid) rocket motors have much flatter thrust vs. time curves than amateur motors. Much care is devoted to the casting of the propellant and to the quality control of the grain and its distribution in professional rockets. This is to ensure specific performance, throughout the burn, and to make sure the thrust level does not go above (or below) certain values so that spacecraft designers will have very definite limits for the "g" forces that will be experienced by the components used in the structure and the body of the spacecraft.

These "g" forces are what we called mushing earlier because they tend to mush (or smash) the components flat. If you've ever seen movies of someone's face in the centrifuge used to train test pilots and astronauts, you understand what mushing is. If you are being accelerated by some force like a car that is speeding up (or slowing down) rapidly, you'll notice that the car seems to be pushing on your back (or your feet) and, sometimes, you have to grab onto something to keep from sliding backward or forward. In rocket and space vehicle design, we have to be very careful to make sure that the stuff we use to build it with is strong enough to withstand the "g" forces.

One time, I got all excited about the performance I could get out of a Thiokol Star 48 SRM (Solid Rocket Motor) on top of a particular configuration of three large booster engines in a "parallel staging" configuration like the Titan and the Shuttle. When I showed the performance curves to the structures experts, they just shook their heads. "Impervium?", I asked. "AAUGH," they said. It turned out that I had only about 100 kg (220 Lbm) of payload. This, and the burnout mass of the Star 48 was the total mass of the "spacecraft" being accelerated by the 10,000 lbf thrust of the engine at burnout. The resulting "g" level (the acceleration) was over 28 "g's" or 275 $m/s^2$. The engine itself was only rated for 12 g's. AAUGH is right.

This is one of the mistakes I made that you don't have to make. Most of the amateur rocket bodies and parts are designed to withstand the "g" forces of all viable combinations of the motors recommended for those rockets. When you start to put together your own stages, made of clusters and superclusters, don't forget to make sure your rocket doesn't get mushed by its own oomph. Always check the maximum acceleration caused by all the forces acting on the vehicle.

So where does this gnome gets its power? Do you have to feed the thing? How much does it weigh? What are the Gnome Union Dues? What about the Gnome's family; don't they object to having their loved one disintegrated everytime he goes to work? That's right Virginia, there is no gnome. So what does the work?

The thrust of a rocket motor is provided by the combustion of fuel and oxidizer, in a partially confined space. The burning propellant (which consists of fuel and oxidizer) becomes very hot, turns into a gas, and is blown out the back of the rocket by the pressure of all the little gas molecules bumping into each other and the walls of the rocket. The energy comes from the chemical combination of fuel and oxidizer. Good rocket propellants are made up of two (or more) chemicals that would much rather be in a different form than they are as unburned molecules of fuel and oxidizer.

Wait a minute, Mr. Author, are you telling us that you're trading in the gnome for some kind of "happiness factor" of the propellant molecules? Is this a gag? No - all chemical reactions can be thought of as the process of the (outer electrons of) molecules going from a particular state to another (of lower energy) in which the molecules are "happier." The unhappiest molecules make the best propellants. (See Fig. 3)

Let's look at one of the best fuel/oxidizer combinations there is - Hydrogen and Oxygen. These are the atoms that make up water, a very stable (happy) substance that covers almost two-thirds of our planet. But when someone has taken the trouble to separate Hydrogen and Oxygen (by using a lot of energy to tear the water molecule apart) the individual atoms are extremely unhappy. They are so unhappy that they form a diatomic (2 atom) molecule to fill their outer electron shells. Even in this state, the combination is extremely volatile and, with the slightest spark or extra heat, the combination will burn and release all the energy that was required to bring the molecules to their unhappy situation. This energy comes out as the speed of the water molecules. The reaction (see Ref. 1) is

$$H_2 + 1/2\ O_2 \longrightarrow H_2O + 57{,}800 \text{ calories/mole of exhaust.}$$

If we work out the speed of the exhaust molecules as in Chapter 5 of Ref. 1, we get an amazing value of 5100 m/s (an *I*sp of 520 seconds)!

If we started with even unhappier molecules like monatomic hydrogen and monatomic oxygen, we'd get an even larger value for the energy (by nearly a factor of 4) and an *I*sp of over 1000 seconds. Unfortunately, monatomic hydrogen and oxygen are so unhappy that they pair up to form the familiar diatomic molecules without any spark or heat being applied so it's almost impossible to store the stuff in a tank until you're ready to launch. An *I*sp of 1000 seconds, combined with today's composite materials would make feasible a (reusable) single stage to orbit launch vehicle that would drastically reduce the funding requirements of launching to low Earth orbit. Dream on, Pollyanna, the engineering difficulties of containing significant amounts of monatomic hydrogen and oxygen are formidable.

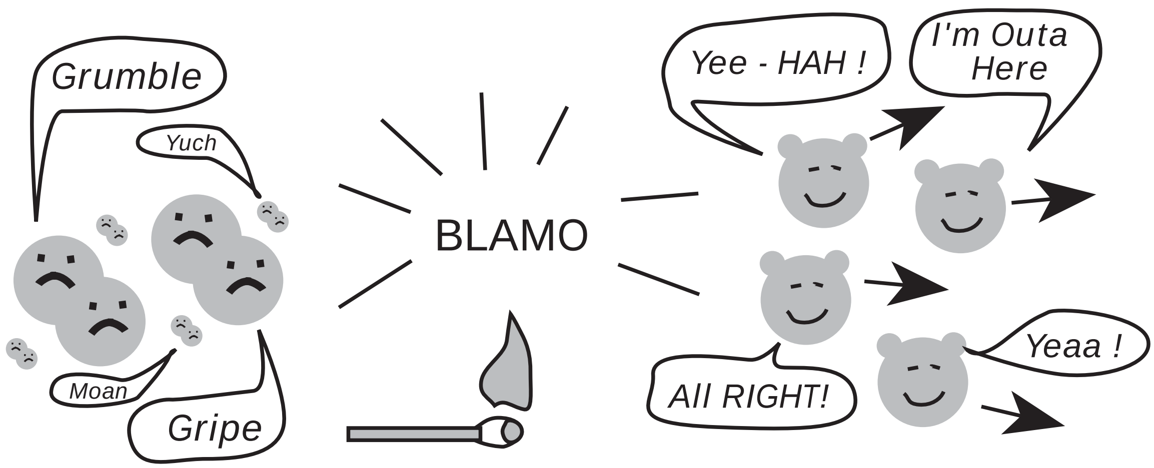


**Fig. 3 Theoretical Aspects of Propellant Molecular Happiness**

And, even if we could do it, we still wouldn't get out all the energy stored in the molecules. Lots of the energy goes into heat and noise. The Shuttle main engines use liquid hydrogen and liquid oxygen as fuel and oxidizer and the whole system (not counting the Solid Rocket Boosters) has a vacuum *I*sp of about 440 to 460 seconds, a far cry from the 520 value we get if we assume perfect energy transfer from the chemical energy stored in the molecules to the kinetic energy of the exhaust molecules.

Amateur rockets (as well as many professional stages) use solid propellant, a grain-like structure made of the molecules of fuel and oxidizer that can be cast into a solid chunk. This process makes it easier to store the propellant and does not require so much additional structural mass for tanks, pumps and engines as liquid rockets. Solid rockets can be made to be much safer than liquid propellants. Thus the outstanding safety record of the (solid) amateur rocket motors. Not all solid propellants are safe, however. Do not try to manufacture your own propellant unless you are a qualified chemist/rocketeer. The amateur motors available today are becoming very sophisticated and are very safe. Why risk blowing yourself up for a few points of *I*sp? If you want better performance, learn to stage the motors that are available.

Now, while we're dreaming, let's examine one more thing that can become a nightmare to the rocket designer. That thing is pressure, caused by the bashing of the recently burned propellant molecules into the walls of the combustion chamber, as if the gnome had taken a bicarbonate of soda, only much worse. As you may have guessed, the gnome is the rocket engine and/or combustion chamber. The gnome's food is the energy stored in the outer electron shells of the fuel/oxidizer molecules, and the structure is what keeps the gnome from exploding in all directions. Gnomes should be unidirectional.

Pressure is force per unit area. In real terms, this means that each tiny little bit of area on the walls of a rocket engine will be subjected to the reaction forces of a whole bunch of very happy molecules bouncing off as they try to get out of the combustion chamber. This causes a great stress on the molecules of the walls of the combustion chamber. If the inter-molecular forces holding the walls together are not strong enough, the walls will split and the rocket will become a fireworks display, like many of the early attempts to design very high performance rockets during the late 1950s and early 1960s.

It is very important that the rocket designer be aware of the forces acting on all parts of the rocket, both external and internal. Not only is the gnome subjected to the "g" forces due to his excellent brick-throwing, he is also subjected to "intestinal" pressure resulting from the bashing of happy propellant molecules against his internal walls. The more bashing, the more structure needed to withstand the bashing, and, usually, the more mass required for the gnome's internal constitution. This all makes sense; the more internal pressure there is to eject the propellant molecules at higher speed, the more strength (and mass) is required for the structure of the engine and its combustion chamber.

Finally, in this chapter, we discuss another pressure - the difference in pressure between the inside of the engine and the air outside. In an earlier discussion, we mentioned the "vacuum *I*sp" of the Shuttle main engines. This term is used to describe an "effective *I*sp" that takes account of the difference between the pressure at the exit plane of the rocket motor and the ambient pressure of the air outside. So why do we speak of "vacuum *I*sp?" The reason is that most rocket motors have a nozzle that allows the escaping exhaust gas to expand just enough to exactly balance the ambient air pressure. The "effective Isp or exhaust speed" is then redefined to account for any difference in pressure between the exit plane of the nozzle and the ambient (surrounding) pressure. When we calculate the "*I*sp" of an amateur rocket motor from its total impulse and its total expended propellant, we are calculating an "effective *I*sp."

Most amateur rocket motors are solid propellant motors that have a clay nozzle. This "nozzle" works OK for a few hundred milliseconds but, then, it is eroded by the forces of the propellant as it burns and forces the exhaust out the back. Some advanced amateur stages have real nozzles that allow the exhaust to expand optimally so as to cancel out the difference between the exit plane and ambient pressure. The "optimal" expansion ratio depends upon the speed of the rocket, the speed of the exhaust, the atmospheric pressure, density, and temperature, and the atomic mass of the propellant molecules. Except for that, it's all pretty simple.

The purpose of this section is not to scare you, but to let you know that there are many more things to be learned about rockets than are discussed in this book. No scientist ever runs out of work. The answer to every question generates more than one new question. The amount of knowledge to be gained in this field is probably infinite. If you think this is a frivolous statement, imagine the following scenario.

Suppose you discover a power source of unprecedented magnitude. Suppose, further, that your new power source is very light, so that you can carry gigajoules of energy in a few grams of stage mass. Ask yourself the question whether it is better to generate a laser beam and shoot it out the back of the rocket, or to heat up a bunch of molecules and blast them out as propellant. Will the laser beam generate any thrust: will it be more than the thrust of a conventional rocket: will anyone ever use it, even if it works? These are the dreams and the nightmares of the innovative scientist and engineer. Do not do these somnambulistic calculations on a computer. Learn to do them in your head. Then your dreams will have substance.

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**References**

1. Pauling, L., “College Chemistry” 3rd Edition W.H. Freeman & Co. , San Francisco, 1964.

2. Asimov, I., “Biographical Encyclopedia of Science & Technology,” AVON Books, New York, 1964.

3. Anon., “Model Rocket Altitude Prediction Charts,” Estes Industries TR-10, Penrose, CO, 1970.

4. Gregorek, G., “Aerodynamic Drag of Model Rockets,” Estes Industries TR-11, Penrose, CO, 1970.

5. Thomson, W. T., “Dynamics of Space Vehicles,” ????,???

Bibliography

Stine, G.H., “Handbook of Model Rocketry,” 5th Edition, Arco Publishing Co., New York, 1983.

Pratt, D. R., “ Basics of Model Rocketry,” Kalmbach Publishing Co., Milwaukee, 1981.

Ashley, H., and Landahl, M., “Aerodynamics of Wings and Bodies,” Dover Publications Inc., New York, 1965

Stine, G.H., “Rocket Power and Space Flight,” Henry Holt & Co., New York, 1957.