FORTUNE EIGHT

Aerospace Industries, Inc.

International Technical Services

Original Lecture: 2002 April 17

MEMORANDUM

To: CMA Class

From: Chauncey Uphoff

Subject: Class Notes for Lecture #13

In Lecture #13, I continued my discussion of the Main Problem of the Lunar Theory (Third-Body Perturbation Theory) and began a presentation of what I call perturbation coupling. I discussed "frozen" orbits as a prime example of perturbation coupling and showed how, by plotting contours of constant averaged (mean) disturbing function, one can determine the phase plane trajectory of the motion. I pointed out that, if the averaged disturbing function does not contain the time explicitly, then the "mean" particle or spacecraft will follow contours of constant mean disturbing function, within whatever phase space we choose to plot the time history of the variables against each other.

For example, if we wish to see the extent of the variations of mean eccentricity versus mean argument of pericenter, we simply plot the variations on a grid of e vs. ω. When we do that, we find regions where the time rates of eccentricity and argument of pericenter are near zero at the same point in the e-ω phase space. That is, there is a "mountain" in the phase space whose peak is a point where neither the mean (singly-averaged) eccentricity nor the mean argument of pericenter change with time. The orbit is said to be "frozen."

This is an example of what I call balancing of forces. In this case, the average (per orbit period) effects of J_2 are being exactly cancelled by the average effects of the third zonal J_3. Actually, the odd zonals J_3 and J_5 are canceling the effects of the even zonals J_2 and J_4. J_5 has only a small, but noticeable, effect upon the position of the frozen orbit in the e-ω phase space. I showed a small program that I use to plot contours of constant potential in the e-ω phase space and pointed out that one must be sure to conserve the axial component of angular momentum (the Delaunay variable $H = (\mu p)^{1/2} \cos i$).

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I then mentioned some practical applications of frozen orbits and a short diatribe on the people who first discovered frozen orbits. R. T. Clapp and his colleague (whose name I regret I can't recall without digging through my old notes) tried to find an orbit about the oblate Earth with constant altitude. They quickly realized that "you can't get there from here" because of the motion of the mean argument of pericenter and mean eccentricity of realistic orbits in an oblate field. But they found that it was possible to keep the altitude fairly constant over the Northern hemisphere and fairly constant (a different near-constant) over the Southern hemisphere by "freezing" the argument of pericenter near 90°.

These orbits are useful for exact repeat orbits (those that repeat their ground-tracks to within a few km) and for synthetic aperture radar missions. These SAR missions benefit from the frozen orbit (and the near-constant altitude over given latitudes) because the radar people don't have to change the pulse repetition frequency very often to obtain the desired resolution of the radar signal. It is not important for the student to understand the details of these complex applications, only that frozen orbits are useful and should not be ignored.

I had intended to show another application of a different kind of frozen orbit – one whose pericenter is frozen by a balancing of the doubly averaged effects of third-body perturbations against the singly-averaged effects of central planet oblateness (J_2). This mechanism is particularly effective for designing "Earth-frozen" lunar orbits that worked well as relay orbits for close lunar satellites so as to get data back from the low lunar orbiter via "bent-pipe" tracking through the frozen high relay orbit.

As I was beginning to explain this application of balanced forces, the power went out in the building. As I was using information and examples stored in my computer, I was unable to show the examples over the projector and was, apparently, dead in the water for this lecture. I asked myself "What can I do in the dark?" I decided to elaborate on the examples I had begun on exact repeat missions and SAR missions and did so for several minutes, but I was running out of things to do without the projector or the chalkboard. Then the lights came back on.

As I had about 30 minutes left to lecture, I decided to give an introduction to solar sailing, the next lecture in the series. As my trusty TAs, Messrs. Stauch and Anderson, were rebooting the projector, the lights went out again.

"*O.K., Herr Gott*," I thought, " *I guess you don't want me to bring up solar sailing yet*." So, I decided to answer the question I had posed early in the lecture: " What are the two most important forces in the solar system?" I had asked the students to think about this during the lecture and, if they wished, to right down their answers on a piece of paper and turn it in at the end of the lecture. But they couldn't even see to write so I said to forget about it. I'm not going to give my answer here because I intend to teach this course again, and I wish to know which of the students can see the connection between balancing of forces (the theme of this lecture) and the two most important forces in the solar system.

So, if you were there, listening in the dark, you got my answer. There are other viable answers, but they can certainly be considered as the generators of the force I consider to be the important force other than the obvious one.

The lecture broke up about 20 minutes early and, as Mr. Stauch and I were talking in the dark, trying to pack up the electronics, the lights came back on for good. Go figure.

Best regards,
Chauncey Uphoff 2002 April 20