Computational Mission Analysis Lecture Notes

2002/1/19 Lecture #2

Conversion of Osculating Elements to Position and Velocity

Given a,e,f,ω,i,Ω , find position and velocity vectors **R** and **V**.

- Calculate the energy E and angular momentum h (per unit mass)
  $E = -\mu/2a = v^2/2 - \mu/r \quad ; \quad h = [\mu a(1-e^2)]^{1/2} = |\mathbf{R} \times \mathbf{V}|$
- Calculate the semi-latus rectum $p = a(1 - e^2)$
- Calculate radial distance $r = |\mathbf{R}| = p/(1 + e \cos f)$
- Get the speed $v = |\mathbf{V}|$ from the energy equation
  $v^2 = \mu\{2/r - 1/a\}$
- Calculate the path angle from $h = rv \cos \gamma$
  or use

$$\tan \gamma = \frac{e \sin f}{1 + e \cos f}.$$

- Express Cartesian components in the orbit plane where x' axis is along the perigee direction and y' points in same direction as velocity vector at perigee. (I think this is how McCuskey did it.)

$$x' = r \cos f \; ; \; y' = r \sin f \; ; \; z' = 0$$
$$\dot{x}' = -v \sin(f-\gamma) \; ; \; \dot{y}' = v \cos(f-\gamma) \; ; \; \dot{z}' = 0$$

- Rotate the primed vectors through the angles Ω, i, and ω in that order, that is:

$$\mathbf{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \; ; \; \mathbf{V} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{bmatrix}$$

where (see e.g. Goldstein. Note: the angles Ω,i,ω are exactly the same as the Eulerian angles φ,θ,ψ in Goldstein's notation).

$$A^{-1} = \begin{bmatrix} c\omega\, c\Omega - ci\, s\Omega\, s\omega & -s\omega\, c\Omega - ci\, s\Omega\, c\omega & si\, s\Omega \\ c\omega\, s\Omega + ci\, c\Omega\, s\omega & -s\omega s\Omega + ci\, c\Omega\, c\omega & -si\, c\Omega \\ si\, s\omega & si\, cw & ci \end{bmatrix}$$

and the leading c's and s's stand for cosine and sine respectively.

Chauncey Uphoff 2002/1/19