FORTUNE EIGHT

Aerospace Industries, Inc.

International Technical Services

Original Lecture: 2002 Feb 6

MEMORANDUM

To: CMA Class

From: Chauncey Uphoff

Subject: Class Notes for Lecture #4

I was somewhat disappointed with my presentation of the optimal (deep-space) staging problem as I tried to revive it (partly from memory) from Thomson's book. My problem was lack of memory and from a change of notation that I made about 15 years ago (1986). The thing I did for American Rocket Company was taken from someone's class notes from an advanced astrodynamics course given at Stanford. I regret that I don't recall whose notes I used but I now recall that the notation was different. I had read Thomson's development in his Chapter 8 previously (c.1973), but could not find my copy of Thomson's book when I wrote the NSTAGE program. So my lecture came across with mixed notation that must have been difficult for the student to follow.

This stuff is hard enough to understand without my having inserted a notation switch during the definition of the problem and the exposition given in my lecture, which was almost directly from Thomson's book. It is my thesis that one should admit his or her mistakes as soon as they become apparent. This is very important in project work and, indeed, in any endeavor in which one's colleagues (or students) rely upon one's inputs.

The part of the lecture that was clear and most important was the discussion of the Lagrange (more properly the Euler-Lagrange) multiplier method and the point that, no matter how many stages are involved, there is still only one constraint and, therefore, only one Euler-Lagrange multiplier. Also stressed was that, in most real problems, the solution for the Euler-Lagrange multiplier requires iteration. Once found, however, the (scalar) multiplier yields the constrained optimal solution to the entire problem which can then be solved either backwards or forwards. The Euler-Lagrange multiplier method is a very

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powerful tool. Euler's name has been dropped from common usage, even though Lagrange acknowledged Euler's earlier work. (See *Oeuvres de Lagrange*, **10**, p. 389).

Also included was Fred Nagle's geometrical analogy for the Euler-Lagrange multiplier method. In this analogy, one draws a topographic map of (say) the Rocky Mountain National Park and the problem is to find the highest point (the top of the highest "mountain") in the park. Well, given the map, that would be easy. But wait, one has to stay on a given "road" (the constraint). Dr. Nagle's analogy is that the Euler-Lagrange multiplier is simply (not so simple to understand) the ratio of the gradient of the constraint to the gradient of the function at the point (in the function space) where they are parallel. This is to say that the constrained optimal solution is the point on the "road" where it is "highest" up the "mountain." A slightly different geometrical interpretation of the Euler-Lagrange multiplier can be found in Donald R. Smith's excellent book "Variational Methods in Optimization." (See recommended reading list on the web-site).

I pointed out that almost nobody uses this method for optimizing the ΔV split between stages of differing Isp and stage-propellant mass fraction (λ', in the more standard notation with which I started). Similarly, I pointed out that this works only in deep space where there are no drag or (gravity loss) effects. That should have made the COV (Calculus of Variations) problem into one with what is called "variable end-points." It took me a long time to realize this when I tried to apply the method to (lower-stage) launch vehicles and amateur rockets. For a long time, I couldn't figure out why I always got answers that didn't make sense. Now I understand what I was doing wrong; I hope some of you get this too. Then you'll know when to use NSTAGE and when not to.

To see the connection between the notation of Thomson's development and the more standard notation used in the program NSTAGE (written in BASIC) on the web-site, one should convert Thomson's "structural factor" β_i to the more standard notation λ'_i for the stage propellant mass fraction of the ith stage. Also change Thomson's exhaust speed, for each stage, to be $c_i = gIsp_i$. I apologize to the class for the notational mix-up.

Best regards,
Chauncey Uphoff, 2002 March 25