FORTUNE EIGHT
Aerospace Industries, Inc.
International Technical Services

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MEMORANDUM

To: CMA Class

From: Chauncey Uphoff

Subject: Class Notes for Lecture #5

In Lecture #5, I repaired the notational confusion of the previous lecture by showing, in the notation of NSTAGE, the ratio of the payload of one stage to its liftoff mass. The notation of NSTAGE is more standard than that of Thomson's development. The ratio of the payload mass of (say) the 1st stage to the lift-off mass of that stage is:

$$m_{PL1}/m_{01} = \{e^{-(\Delta V1)/c1}) / \lambda'_1 - [1/\lambda'_1 - 1]\},$$

where λ'_i is the stage propellant mass fraction for the i^{th} stage and c_i is the exhaust speed ($gIsp_i$) for that stage. Here, g is the acceleration due to gravity at Earth's surface (9.8066 m/s^2) and this is simply a units change. Use the same value of g no matter where, in the Universe, you burn the rocket.

Now, with Thomson, note that $m_{PL1} = m_{02}$, which is to say that the lift-off mass of the 2nd stage is the payload of the 1st stage. Therefore, we can chain the mass ratios (and take the natural logarithm), as Thomson did, to get the payload of the final stage in terms of the initial mass of the total stack.

It was pointed out that stage propellant mass fraction is defined as the mass of the useful propellant, m_p, of a stage to the mass of the useful propellant plus the mass of the inerts of that stage. That is: $\lambda' = m_p/(m_p + m_I)$.

One must take care in defining m_I for a given stage; m_I should be defined as everything that is dropped between burnout of the lower stage and ignition of the stage above it. This inert

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mass includes the tanks, engines, unburned propellant, residual pyro mass, and any ham sandwiches left in the lower stage by the people who nailed the upper stage to the lower stage. The "ham sandwich" factor is very small in commercial rockets, but there is an occasional extra bolt or (Heaven forbid) a pair of pliers, or other forgotten tool, that will not only add to the magnitude of m_I, but may also rattle around so violently, during ascent, as to destroy the entire vehicle. If you get into launch vehicle performance and optimal staging, you'll have to think about these things carefully.

Also included in Lecture #5 was a discussion of the BASIC program, NSTAGE, that finds the optimal ΔV split between N-stages, of a multi-stage vehicle, having different values for c_i and λ'_i whose objective (constraint) is to add a given total ΔV_T (= $\Sigma \Delta V_i$) to the upper stage payload. The BASIC program NSTAGE has a "successive approximation" iteration, that always seems to work, to find the Euler-Lagrange multiplier that satisfies the constraint. If this iteration doesn't work, it's probably because I've done something stupid in specifying the order of the "good" and "bad" stages. NSTAGE has a number of initial guesses (and tests of those guesses) for ensuring that the Euler-Lagrange multiplier will be properly determined, if it exists in a useful function space, for the particular problem posed.

I pointed out that one must be careful in assigning adaptors and fairings to various stages of (stacks) of rocket vehicle systems. For example, if the stage 1/stage 2 adaptor is "charged" to the stage 1 inerts, then the inerts are the stage inerts plus the upper stack adaptor between stage 1 and all the stages above. This is a typical mistake made by novice astrodynamicists and bureaucrats who wish to show the "great" performance of their proposed vehicle. I warned the students to always ask (of the providers of launch-vehicle performance charts), "Does that 'payload' include the adaptor?" Trust me, it always does, and it's something you're going to have to throw away or carry with the next stage. In either case, the adaptor should be included in the inert mass (m_I) of one stage or the other.

Later, in the lecture, I discussed the more complex aspects of the Calculus of Variations and how the "Heavy" programs of optimization use the Pontryagan Maximum Principle. These programs give an automatic optimal coast period in (e.g.) interplanetary transfer trajectories and show exactly where to apply impulses (or arcs of low thrust) so as to satisfy the necessary and sufficient conditions of constrained optimal transfer. These theories and programs use what is called "Primer Vector Theory" but they still require good guesses for the (equivalents of) the Euler-Lagrange multipliers in multiply constrained dynamical problems.

As problems become more complex, multi-dimensional, and multiply constrained, such as those for launch vehicles, the optimization problem is usually solved with what are called "direct" methods or hunting procedures. One of the most powerful of these is the "steepest descent" with minimum step size. Here, minimum step size refers to the change in the control variable(s) required to achieve a certain "gain" in the function to be maximized (or minimized). For launch vehicles, this is usually useful payload delivered to a given orbit. One of the major difficulties with these methods is that one often takes too big a step, trying to find the extremal value of the function, and steps right over it into a region of the function space far from the extremal. Steepest descent (often called steepest ascent) refers to selection of the gradient of the function as the direction in which to move to achieve a desired "gain." But remember, one must stay on Fred Nagle's "road" for constrained optimal problems. In most cases, there are many "roads" to stay on. In over-constrained problems, the "roads" never cross each other anywhere in the desired function space.

The literature is filled with papers on constrained optimization and, as the details of this subject are far beyond the scope of this course, I shall simply refer the student to the literature and, in particular, to Donald R. Smith's wonderful book which, I have just realized, is not on the reading list for this course as I said in Lecture Notes#4. It should be. Anyone who wishes to understand this stuff is encouraged to start with Professor Smith's book. The reference is:

Smith, Donald R., "Variational Methods in Optimization," Prentice-Hall, 1974.

Best regards,
Chauncey Uphoff, 2002 March 28