FORTUNE EIGHT
Aerospace Industries, Inc.
International Technical Services

2002 February 4

MEMORANDUM

To: CMA Class
From: Chauncey Uphoff
Subject: Conic Sections

My good friend and colleague, Dr. Frank Janssens of Mechelen Belgium, is monitoring the CMA course on the Internet. Frank is a real professional dynamicist and is very interested in Physics, History, and Geometry. He has made some valuable comments and has tried and solved some (but not all) of the problems. He too, had some trouble with the foci and the engineering drawing problem. So, even the pros have some trouble with these problems; don't worry that you can't get them immediately. These are some of the most subtle problems I can remember and they're in here for a good reason. You'll understand better, at the end of the course, that I don't include them just to make the course fun; that's only a superficial reason for the puzzles. All of Physics is the ultimate puzzle and each one you solve generates new puzzles you hadn't thought of. Learn to live with it; it's part of a natural law that the politicians can't change. Have you ever broken one of Newton's laws?

Frank has sent me the reference to Apollonius's 8-volume set of books on "Conic Sections." Frank has been looking for the source of the Latin expression *latus rectum*. See Danby's (revised) 2nd edition (pp. 413-418, Appendix A). There, Danby has the following reference to the work of Apollonius as follows:

> "The line OD, being an axis of symmetry, is an axis of the conic. The minimum value of r occurs when $\nu = 0$, and the maximum, if it exists, when $\nu = \pi$. The chord through O, perpendicular to OD, is called the latus rectum and has the length $2p$. p is often referred to as the *semilatus rectum*. (In the constructions of Apollonius. (c. 200 B.C.) a rectangle was formed based on a line tangent to the conic and sticking out from the cone; this line has length $2p$. It was called *ορθια*, meaning erect [there's a funny little grave or caret over the Greek o that probably indicates the word is pronounced 'órtia' with a soft Russian t], and the original Latin translation was *latus erectum*. The phrase used today is a corruption of this.)"

Thus, Danby provides a reference to Frank's question about the source of *semilatus rectum* through Apollonius who must have known about the spheres that define the foci (or focus for the parabola) of the conic sections.

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One needs to see Danby's pictures A.2 and A.3 to see the connection. Frank, this is probably the connection you need to characterize the orbits of particles with equal angular momentum (per unit mass) without having to draw an unintelligible diagram in the plane. It comes from somewhere in the 8 books of Apollonius. The little sphere is there because Apollonius understood (and/or) defined the directrix. Another possibility is to take inclined sections of an elliptic paraboloid or parabolic hyperboloid. Try to make the distance from the focus to the surface of revolution constant perpendicular (erect) to the direction of the major axis; that might work.

Dr. Janssens has also provided a connection to what sounds like a good book. Frank wrote:

> " . . . David Wells' 'The Penguin Dictionary of Curious and Interesting Geometry' (1991) giving the property of the spheres as Dandelin's theorem. Dandelin was a Belgian professor. The theorem dates from around 1820 -30 ? Do your sources refer to this ? (I expected that property to [have] a longer history) For the constant p-family, I wonder if something can be achieved with a skew cone, but this is only vague."

Perhaps Apollonius included only the sphere nearest the apex of the cone and Dandelin showed the other one tangent to the empty focus. Who knows? I'm surprised that Descartes didn't address this problem; perhaps he did.

The point, here, is that my closest friends and colleagues are interested in the problems posed in this course and that they can't get all of them without hard mental work. That should tell the student that there are important things to be learned by the puzzles and questions. Do not ignore the History of these ideas; it is far more important than the Establishment would have us believe. Nothing comes from nothing.

Also note that "Plane and Solid Analytic Geometry" by Osgood and Graustein (the MACMILLAN Co. , 1920, 23rd printing 1956) is the only other place I've seen this definition of the foci of conic sections (prior to Danby's 2nd edition.) That should tell some of us that it's important. Unfortunately, Osgood and Graustein were Harvard professors and made their book almost a "ding an sich." There are very few historical references and those only in footnotes. But the book has almost every (left-brain) thing one would want to know about analytic geometry with proofs and very proper cross-references to previous developments in their book.

Best to all,
Chauncey Uphoff
2002 February