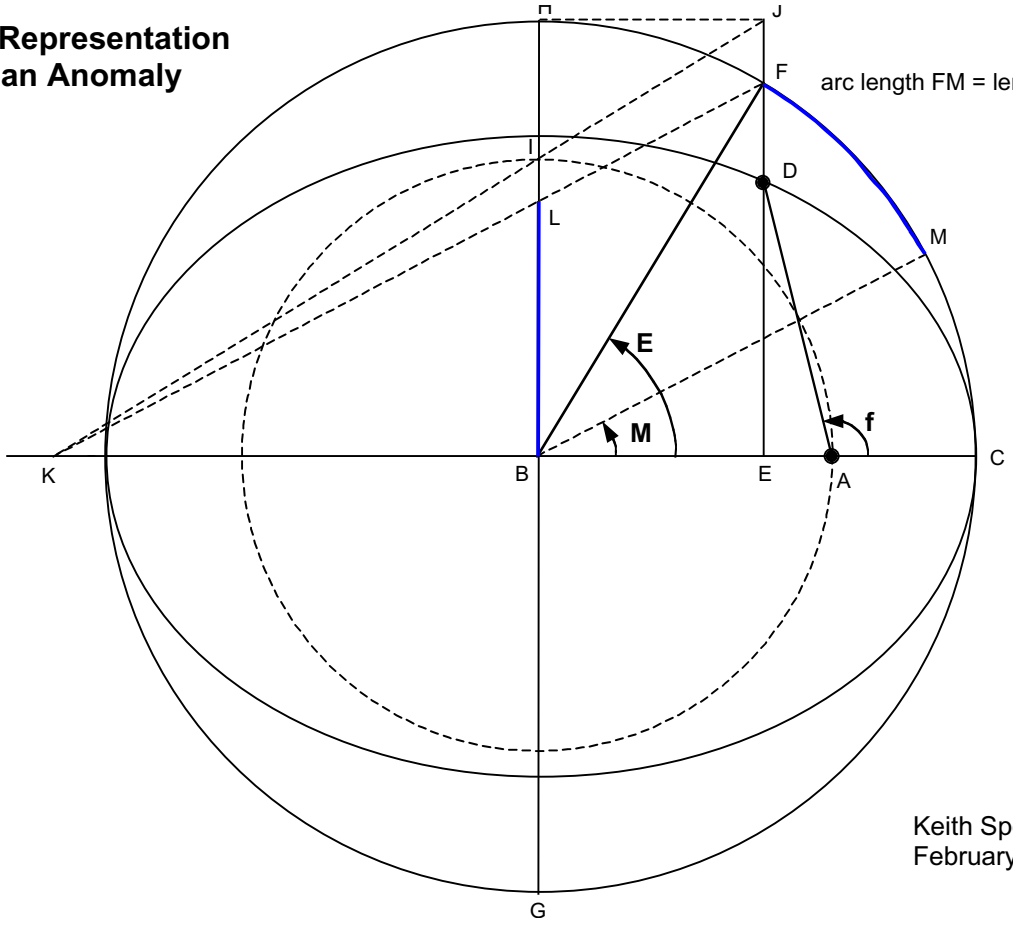
Graphic Representation of Mean Anomaly



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This picture shows an orbit with an attracting focus at A and it s auxiliary circle with a center at B. The periapsis is located at point C. An orbiting body is located at point D. The true anomaly, f, is defined by angle CAD. A vertical line EF is drawn through D. Angle FBE defines the Eccentric Anomaly, E. Vertical line GH is drawn for reference.

Now, we determine the Mean Anomaly, M, from Kepler s equation:

$$M = E - e*\sin(E)$$

Eccentricity, e, can be defined as the length between the focus and the center, AB, divided by the length of the semi-major axis ,BC (AB/BC). A circle is drawn intersecting point A with a center at point B to intersect line GH at point I such that AB equals BI. Since BH equals BC, we get the following representation of eccentricity.

$$e = BI/BH$$

The sine of E is equal to the ratio of the length of the opposite side over the hypotenuse (EF/BF). Horizontal line HJ is drawn so that BH equals JE. Since BF equals BH, we now we have the following representation for sine of E.

$$\sin(E) = EF/EJ$$

Now a line is drawn connecting point I with point J and intersecting an extension of the semi-major axis at point K. Another line is then drawn joining K and F to intersect GH at point L. Now we have 2 sets of similar triangles; KEJ is similar to KBI, and KEF is similar to KBL. Therefore EF/EJ equals BL/BI. This leads to the following representation for e*sin(E).

$$e*\sin(E) = BI/BH * BL/BI = BL/BH$$

Now, we substitute BH with the length of the radius of the auxiliary circle, rac, so e*sin(E) = BL/rac. Therefore, the length BL/rac is equal to the angle between the Eccentric Anomaly and the Mean Anomaly in radians. The arc length of this angle along the auxiliary circle is then equal to this angle times the radius, rac. Therefore, the arc length of the difference between the Eccentric Anomaly and the Mean Anomaly is equal to the length BL.

Point M is then added in the negative arc direction such that arc length FM equals length BL. The Mean Anomaly is then angle CBM. Note that in quadrants 3 and 4, BL points in the opposite direction and could be treated as a negative number to give results with the Mean Anomaly larger than the Eccentric Anomaly.

As the spacecraft moves in its orbit, line EF moves with it. This causes point F to move up and down. Points L and K move around with it as point I stays fixed. Line JE could be projected left or right in order to easier locate points K and L when it gets too close to line GH.