FORTUNE EIGHT

Aerospace Industries, Inc.

International Technical Services

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MEMORANDUM

To: John Evanyo, BATC
From: Chauncey Uphoff, F8

Subject: Stability of PQ Earth Capture Orbit

This memorandum contains a discussion of the stability of the Earth Capture Orbit (ECO) to be used for all options in case we cannot show that the probability of accidentally entering Earth's atmosphere is less than 10^{-6}, the PQ requirement. Discussions of the work of Lidov are included along with a description of how Lidov's analysis has been used to "stabilize" the highly perturbed orbits of IMP-H and IMP-J. The word stabilize is in quotes because these large Earth orbits are subject to resonance perturbations not included in Lidov's analysis. For IMP-H and –J, however, it was found possible to thread through the resonances and satisfy the mission objectives. This can be done by making minor adjustments to the semi-major axis of the orbit and by adjusting the inclination and the argument of perigee (wrt the Earth-moon plane).

Lidov's Work:
In the early 1960s (Ref. 1), Lidov found solutions to the doubly averaged equations of motion in the circular restricted three-body problem. In particular, Lidov obtained analytic integrals to the doubly averaged equations (one of them elliptic) and showed that the maximum eccentricity of the perturbed orbit could be described in terms of the two algebraic integrals

$\mathbf{C_1 = (1 - e^2) \cos^2 i}$, **and** $\mathbf{C_2 = e^2 (2/5 - \sin^2 i \sin^2 \omega)}$, viz:

$$e_{\max} = \sqrt{\frac{1}{2}\left\{\left[1-\frac{5}{3}\left(C_1+C_2\right)\right]+\sqrt{\left[1+\frac{5}{3}\left(C_1+C_2\right)\right]^2-\frac{20}{3}C_1}\right\}}\ .$$

In the above, e, i, and ω are the eccentricity, inclination, and argument of perigee of the doubly averaged orbit; the latter two variables are referred to the plane of the disturbing (third) body about the central primary (the Earth-moon plane in our case). Note that, if C_1 is 0, $e_{max} = 1$, which equation implies that the S/C will crash into the Earth or escape the Earth-moon system (or maybe crash into the moon).

The 1st integral above is simply an expression that the polar component of angular momentum is constant, a fact that is obvious from the doubly averaged equations because they do not contain the longitude of the node on the right hand side. In this case, the longitude of the node is said to be an ignorable variable.

It should be mentioned that Rick Williams and Jack Lorell obtained these same equations independently in the early 1960s (c. 1966; see Ref. 2) by different methods. Williams and Lorell also showed that the doubly-averaged equations could be solved in terms of elliptic integrals.

Lidov's equations, without the effects of oblateness are simply:

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$$\frac{da}{dt} = 0$$

$$\frac{de}{dt} = \frac{C}{2} e\sqrt{1-e^2}\sin^2 i \sin 2\omega$$

$$\frac{di}{dt} = -\frac{Ce^2}{4\sqrt{1-e^2}}\sin 2i \sin 2\omega$$

$$\frac{d\omega}{dt} = \frac{C}{\sqrt{1-e^2}}\left[\left(\cos^2 i - 1 + e^2\right)\sin^2\omega + \frac{2}{5}\left(1-e^2\right)\right]$$

Plus an equation for the motion of the node that we won't need here. In the above,

$$C = \frac{15}{4}\frac{n'^2\mu'}{n}\left(1+\frac{3}{2}e'^2\right),$$

where n' is the mean motion of the disturbing body about the central primary, n is the mean motion of the spacecraft in its orbit, μ' is the ratio of the mass of the disturbing body to the mass of the disturbing body + the mass of the central primary; in our case μ' = GMM/(GMM+GME) where GMM and GME represent the gravitational constants of moon and Earth respectively. The mean motions may be expressed as radians per second or degrees per day or degrees per year, whichever is convenient. e' is the eccentricity of the orbit of the disturbing (3rd) body (0.055 for the moon).

Note that these 4 equations are all we need to solve for the time history of the eccentricity, inclination, and argument of perigee. Indeed, the time history of the three variables can be expressed in terms of Jacobian elliptic functions. But the equations are so easy to integrate that I simply use a numerical routine with a step of about 10 or 20 days and get the time histories as a table or graph. From those tables/graphs, I can see the period of the motion. While the period of the motion depends upon the ratio of n'^2 to n, the amplitudes of the variations are independent of the values of n, the mean motion of the spacecraft, or the mean motion, n', of the disturbing body about the central primary. This independence of the amplitude variations from those of the mean motions is very different from the assumptions of the old lunar theory and is one of the major results of Lidov's theory.

It should be noted, however, that Lidov's theory does not include the effects of resonances of the spacecraft motion with that of the moon and, for some orbits, these can be quite large. Also, the moon's orbit does not cooperate by standing still and moves under the influence of the Sun's perturbations. Even so, the long-periodic perturbations are quite strong and can be used to stabilize the orbital motion of apparently unstable orbits for long periods of time. Certainly long enough to launch an Earth Orbital Sample Capture Vehicle, rendezvous with the sample, and bring it home.

Idealized Time Histories:
Below are some numerical integrations of Lidov's doubly averaged equations given above. It was mentioned above that the time history of the eccentricity can be written as (the square of?) an elliptic function of time (Jacobi's dn) which is a function of time and a parameter (k) that tells us how close to circular functions we are and how far from hyperbolic functions. For example, if the value of the integral C_2 is zero, the modulus of the elliptic function is 1 and the time history of the eccentricity is the inverse square (as I recall) of an hyperbolic cosine [e = Constant / $\cosh^2(n''t)$], where n'' is the mean motion of the long-term variations in e, i, and ω. Furthermore, the Sun will be doing the same kind of thing to the orbit but the long-periodic solar perturbations are much weaker (a factor of about 180) than those of the moon.

Thus, the long-periodic lunar perturbations are extremely powerful for use as a stabilizing "pressure" on the orbit to keep it from crashing into the Earth because we can choose the eccentricity, inclination, and argument of perigee to maximize the time during which the S/C orbit will be nearly circular.

Fig. 1, below is an example of such an orbit, whose eccentricity will never increase above that of the initial orbit and has a period (of the eccentricity variation) of about 11 years. If we could expect the idealized disturbing function (Lidov's equations), the period would be infinite for values of $\omega \pm 90°$ with an inclination of 39°.23152 (where $C_2 = [2/5 - \sin^2 i \sin^2\omega] = 0$). Fig. 2 shows the time history of the eccentricity for the same orbit but with an initial inclination of 40° with respect to the Earth-moon plane. Note the difference in period of the long-periodic variations.

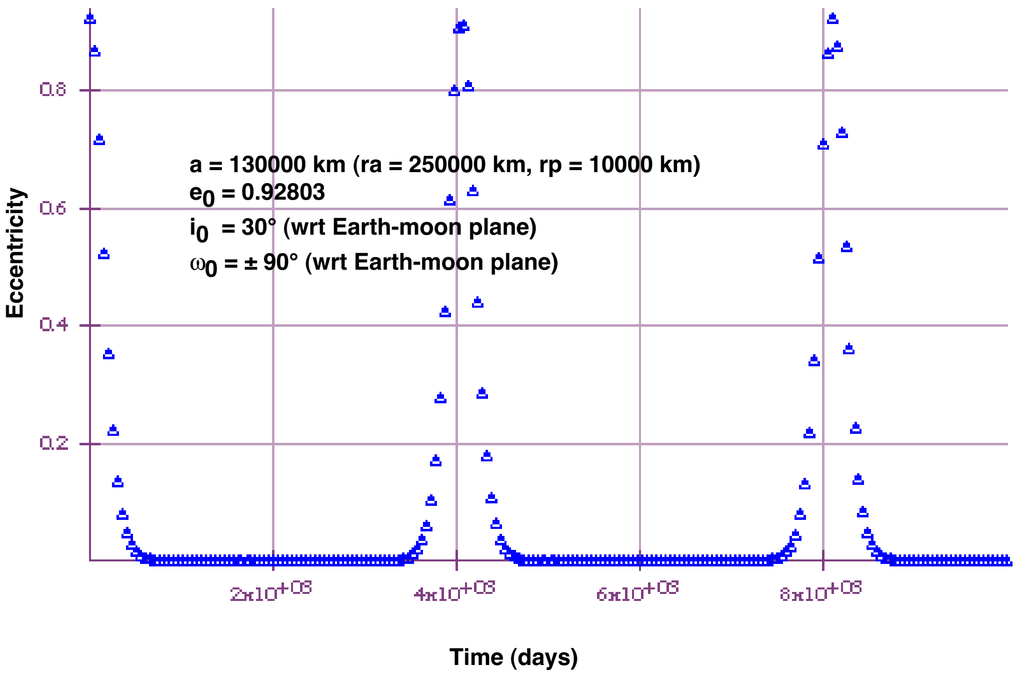

Fig. 1 Idealized Time History of Eccentricity for HEEO at 30° wrt Earth-moon Plane

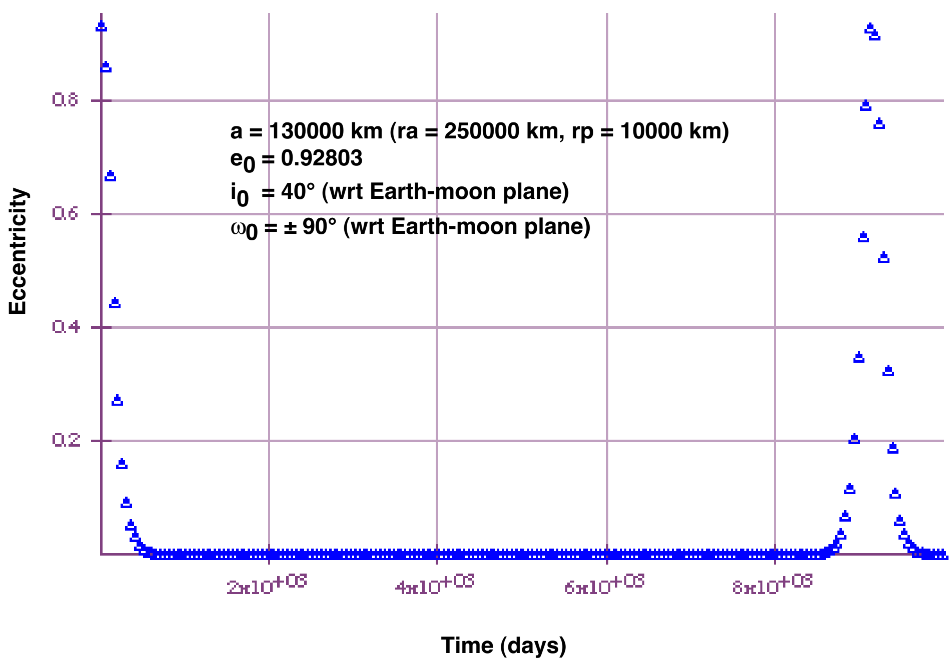

Fig. 2 Idealized Time History of Eccentricity for HEEO at 40¡ wrt Earth-moon Plane

Figs. 1 and 2 do not show what will really happen; they show only the enormous downward pressure on the eccentricity of the orbit (due to the lunar long-periodic perturbations) for orbits in this region. There will be resonance effects, solar gravitational perturbations, and some effects of oblateness for the low-perigee portions of the trajectories. Perhaps we can find a way to use these perturbations rather than try to fight them. Indeed, there may be ways to combine these perturbations with the motion of the moon's orbit plane to obtain very long lifetime (millions of years) orbits. If you're interested, ask me about Bagby's moons. We can't do this within our budget but it might be a real nice Ph.D. thesis for Jason or somebody at CU.

Jason, Scott, and Mike – we'll need to show that these values of inclination and argument of perigee (wrt the Earth-moon plane) are achievable at Earth-return in the 2013 Mars-Earth opportunity. I think we can always do it but I haven't the time to prove it.

And I know I've changed the C3 of the ECO from –2 to –3 km^2/s^2. Don't confuse me with facts; we can always stage the solid for ECO insertion or bring SEP in at C3 = +1 instead of +2. Nyet problyem.

References:

1. Lidov, M.L. "Evolution of the Orbits of Artificial Satellites as Affected by Gravitational Perturbation from External Bodies" AIAA Jrn. Vol 1, No. 8 Russian Supplement August, 1963. (Original in Russian, 1961).

2. Williams, R.R., and Lorell, J., "The Theory of Long-Term Behavior of Artificial Satellite Orbits Due to Third-Body Perturbations," Jet Propulsion Laboratory, Technical Report 32-916, February 1966.

Distribution: Ball MSR Team