FORTUNE EIGHT
Aerospace Industries, Inc.
International Technical Services

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MEMORANDUM

To: Whom It May Concern

From: Chauncey Uphoff

Subject: The History of the Term C3

During the first 20 years of my career, I was asked at least 30 times, whence came the terminology C3 (or C_3) to describe twice the energy (per unit mass) of a spacecraft in orbit about a planet whose gravitational potential is described by an inverse square law of the distance from the center of the planet. In those days, I didn't know and didn't care. During the last 20 years, I have been asked the same question at least 20 times. I'm getting very embarrassed for not being able to answer this question – after all, I'm supposed to be one of the big irons in celestial mechanics; if anyone knows this, I should. But I had to call almost everyone still alive who remembers the work that was done in the late 50s and early 60s to find the answer. I should have remembered, but I didn't, the exposition in Forest Ray Moulton's classic book "Celestial Mechanics" (Original Copyright 1914, The MacMillan Company). The copy I used was the 15th printing (1962). As it may be embarrassing for some of the people who did not know the source of this terminology, I'll leave their names out of this memo. The list would include almost everyone I know, including myself.

The good guys, (not to say that the other people were bad guys; they just didn't know this piece of history) Lou Friedman (who told me to call Vic Clarke), Vic Clarke, and Myles Standish (who kindly mailed me his copy of Moulton) got me to the source of the terminology. In the late 50s and early 60s, there were few readily available sources of information on 2-body orbital dynamics, and even fewer on the 3-body problem (still unsolved to this day, even in the constrained formalism of the Circular Restricted Three Body Problem).

In any case, it fell to Vic Clarke to write up the solution of the two-body problem as a JPL Technical Memorandum (they called these publications TRs, Technical Reports for external publication) and Vic followed the development of Moulton and used Moulton's terminology. So the "official" terminology for "Energy per unit mass" came out as C3/2, almost directly from Moulton's book. Because this terminology was "cast in concrete" in an official JPL document (TR-32-30, 1960 July 26, "Design of Lunar and Interplanetary Ascent Trajectories"), it became the basis of many documents and presentations that showed launch vehicle performance as a function of C3. Check out any launch vehicle performance

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document for transfer from Earth to Mars or Earth to Anywhere. C3 is simply $v^2 - 2\mu/r$. Note that C3 = $Vinf^2$ for trajectories with energy greater than zero. In the above, $\mu = k^2M$ in Moulton's (Gauss's) notation, and M is the total mass of the two bodies. k^2M is often called GM of a planet or central body and $G = k^2$ is the universal (Gaussian) gravitational constant named for Karl Friedrich Gauss.

I have above referenced the JPL document that used Moulton's notation for C3. But this story, like most, gets deeper. Gary Flandro told me of a course given under the auspices of NACA (not yet NASA) in 1958, taught by Dr. Howard Seifert, and later published as a compendium called "Space Technology," edited by him. This course was one of the several avenues I pursued (at Gary Flandro's suggestion) in the quest for the origin of C3. Dr. Seifert encouraged the students to use Moulton's book and it may have been that some workers in the very young Earth-orbital astrodynamics field began to use the expression "c_3" to represent twice the total energy (per unit mass) of the particle or spacecraft, after having read Moulton's exposition of the solution to the two-body problem. However, Professor Flandro was unable to find explicit references to the term C3 (or C_3 or c_3) in his notes from Seifert's course; it was almost certainly the JPL document (and Vic Clarke's write-up) that propagated the term C3 because, in those days, Caltech's JPL was considered by many to be the source of all knowledge.

Moulton's analysis started with angular momentum (per unit mass) of the particle as $c_1 = [\mu a(1-e^2)]^{1/2}$. c_2 is a variable related to the time of periapsis passage (similar to the mean anomaly at epoch); c_3 is twice the energy (per unit mass) of the particle; c_4 is π + the argument of periapsis in the plane of the orbit, measured in the direction of spacecraft motion, from the ascending node of the orbit on the reference plane to the position of the periapsis. The other two constants of the motion have to do with the orientation of the orbit plane with respect to the reference plane. These quantities are usually referred to as the inclination and the right ascension (longitude if given in degrees instead of hours) of the ascending node. See Article 87 and 88. of Moulton's book (pp. 146-149) for the relationships of the integration constants to the classical orbital elements.

I am grateful to my friends and colleagues to have this question at last cleared up. The source of the term "C3" is to be found in Moulton's book in the section called "Problem in the Plane." The relevant sections are in Chapter V of the 15th printing called "The Problem of Two Bodies." Somewhere, Moulton's c_3 became capitalized and the 3 was (for some time and then) no longer a subscript. Moulton does not say whether he followed someone else's notation (I doubt it) but he refers the reader to Tisserand's *Méc. Cél.*, vol. I, chapters VI and VII, in an excellent historical sketch and bibliography at the end of (Moulton's) Chapter V. Vic Clarke probably used the 14th printing (because the JPL document came out in 1960) and the pages and sections may be different although this is unlikely after 13 prior printings.

Now, through Kepler's equation, $M = E - e \sin E$, where M is the mean anomaly, E is the eccentric anomaly, and e is the eccentricity of the orbit, we have a one-to-one correspondence between the osculating orbital elements and the instantaneous position and velocity of the particle or spacecraft.

Professor Newton was well aware of the relationship between the time (Mean anomaly) and angular position in the orbit (true anomaly or $\theta - \omega$in Moulton's notation). Isaac Newton was the first to truly understand the dynamics of orbital motion; he discovered the principles from which we have now gained access to the Universe. If I refer to "Newton's Laws," please don't ask "Wayne or Huey?".

Newton's laws are the foundation of our technology and our (limited) Freedom in the United States of America. If you think Freedom comes from some kind of political or religious mechanism, come and see me; we can fight about it "without punching each other in the nose." Freedom comes from knowledge, as Professor Galambos has articulated. If you want to discuss this, bring many good books, knowledge of history, and extremely concise arguments; then, we might get somewhere.

One final note may be appropriate for historians; Vic Clarke told me this evening that he was a group supervisor in JPL Section 312 at the time he wrote up the Moulton solution to the "Problem in the Plane." At that time, the section manager was Dr. Clarence Gates (known as "Johnny" to most of us who worked for him) and his assistant section manager was Tom Hamilton. Vic told me he decided to include the write-up of the problem in the plane on his own. However, the final version of TR-32-30 would have been reviewed by Hamilton or Gates or both and probably others.

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